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PAPER

The red shifts of pulses of light which are emitted at a point on the surface of a sphere and at a point inside of the sphere comprising electrically counterpoised dust with constant uniform density as observed by an observer in a large distance away in the exterior region

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A sphere, comprising a special kind of matter, with electrically counterpoised dust in which all the elastic forces have been cancelled out has been considered.

A static spherically symmetric solution to Einstein's field equations has been found using a new set of boundary conditions. In introducing these new boundary conditions, we assume that the radial coordinates in and out of the sphere need not be the same and we are guided by the notion of what may be called proper distances and proper times of two observers on either side of the sphere .In these new boundary conditions we replace ordinary partial derivatives by generalized partial derivatives in curvilinear coordinates.
Then the solution takes the form

$$ds^2 = \frac{1}{\left(\theta\left(\frac{r}{l}\right)\right)^2} c^2 dt^2 - \left(\theta\left(\frac{r}{l}\right)\right)^2 \left(dr^2 + r^2 d\Omega^2\right) 0 \le r \le a$$

$$ds^2 = \frac{1}{\left(1 + \frac{A_2}{R}\right)^2} c^2 dT^2 - \left(1 + \frac{A_2}{R}\right)^2 \left(dR^2 + R^2 d\overline{\Omega}^2\right) A < R$$

where $A_2 = -\frac{a^2}{l}\theta'\left(\frac{a}{l}\right)$, $\theta\left(\frac{r}{l}\right)$ is the solution of the Lane-Emden equation $\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right) = -y^3$, $r = lx$, l is a constant of dimension length , a is the coordinate radius of the sphere.
In our approach $r = a$ in the matter-filled region corresponds to $R = A$ in the region without matter, outside the sphere.

The red shift of a pulse of light emitted at a point on the surface of the sphere as observed by an observer who is at a large distance in the exterior region of the sphere is calculated. This valueequals to $\left(\frac{\theta\left(\frac{a}{l}\right)}{\theta\left(\frac{a}{l}\right) + \frac{a}{l}\theta'\left(\frac{a}{l}\right)} \right)$ when the observer is at infinity.

The comparison of this value with the value for the red shift obtained using the metric derived using the standard (Lichernowicz) boundary conditions which says that the metric coefficients and their partial derivatives are continuous across the boundary of the sphere when the observer is at infinity is also done. It is shown that the values obtained for the red shifts are the same irrespective of the boundary conditions used.

The red shift of a pulse of light emitted at a point inside of the surface of the sphere as observed by an observer who is at a large distance in the exterior region of the sphere is also calculated and it is shown that the value obtained is different from the value obtained using the metric derived using standard (Lichernowicz) boundary conditions.

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