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Poster

Valuation of options in Black-Scholes Model using finite difference methods

A benchmark mathematical model for the description of financial derivatives was introduced by Fischer Black and Myron Scholes in (1973, [1]) and equation was simplified further by introducing $Z = \ln S$ (Brennan and Swartz, 1978, [2]) and resulting equation is given by

$$\frac{\partial f}{\partial t} + \left(r - \frac{\sigma^2}{2}\right)\frac{\partial f}{\partial Z} + \frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial Z^2} = rf \qquad (1)$$

where,

S : price of the underlying asset, r : risk free interest rate, t : the time,

σ : volatility of the underlying asset, f : price of the derivative

In the absence of assumption free analytical method to obtaining the solution of the full model, various numerical algorithms are used. This work is mainly focused to analyze three finite difference methods namely Forward Time and Centred Difference (FTCS), Backward Time Centred Difference (BTCS) and Crank-Nicholson (CN) .

It was found that both implicit FTCS and explicit BTCS and implicit CN schemes are consistent and explicit BTCS scheme is conditionally stable under

$$\frac{\Delta t}{\Delta Z^2} \leq 1 \qquad (2)$$

and both implicit FTCS and implicit CN schemes is unconditionally stable.

Hence all the schemes are convergent in the view of Lax-Richtmayer equivalence theorem. Finally these algorithms are implemented using MATALB and the convergence properties of the schemes are shown by numerical experiments.

Keywords: **Financial Derivatives, Black-scholes Equation, Finite Difference Methods**

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