4.25 On the Schwarzschild singularity

R.L.R.A.S. Rajapaksha, Nalin de Silva
Department of Physics, University of Kelaniya.
Department of Mathematics, University of Kelaniya.

ABSTRACT

The Schwarzschild metric $ds^2 = \left(1 - \frac{2m}{r}\right)c^2 dt^2 - \frac{1}{\left(1-\frac{2m}{r}\right)} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$ appears to behave badly near $r = 2m$, where g_{tt} becomes zero, and g_{rr} tends to infinity[1]. There is a pathology in the line element that is due to a pathology in the space-time geometry itself.

The worrisome region of Schwarzschild metric, $r = 2m$, is called the "event horizon". It is also called the "Schwarzschild singularity"[1].

There are many coordinate systems that have been found to overcome the Schwarzschild singularity[1,2]. By using the Schwarzschild metric in Schwarzschild coordinates, in Eddington-Filkelstein coordinates and in Kruslal-Szekeres coordinates, we have obtained some expressions for geodesics to check the behavior of a test particle at $r = 2m$, and in the two regions, the region outside $r = 2m$ and the region inside $r = 2m$.

We have shown that in all the coordinate systems it is consistent to take $\frac{dr}{ds} < 0$ when $r > 2m$ and $\frac{dr}{ds} > 0$ when $r < 2m$. The coefficient of dr^2 becomes negative when $r < 2m$, making r a time like coordinate in that region. Thus r has to increase in this region. Further $\left|\frac{dr}{ds}\right|$ becomes greater than c, the speed of light when $r = 2mk$, where k is a constant that depends on the initial condition, in the case of Schwarzschild coordinates and Eddington-Finkelstein coordinates, and there is jump at $r = 2m$, in $\frac{dr}{ds}$ from $-cl$ to cl, where l is a constant.

These results suggest that once the particle crosses the event horizon at $r = 2m$ it tends to remain there as $\frac{dr}{ds} > 0$, when $r < 2m$ in all the three coordinate systems. A transformation of coordinate does not change this fact and we may suggest that the particle does not cross the event horizon, making it more than a mere coordinate singularity.

The fact that $\left|\frac{dr}{ds}\right|$ becomes greater than c in the neighborhood of $r = 2m$ at least in two coordinate systems also suggest that the particle is changed physically around $r = 2m$.

Hence we may say that the singularity at $r = 2m$ is a physical singularity and not merely a coordinate singularity.

References:

1. Misner Charles W., Thorne Kip S., Wheeler John Archibald; *Gravitation*; W.H. Freemon and Company, San Francisco (1970)

2. Hartle James B.; *Gravity, An Introduction to Einstein's General Relativity*; Dorling Kindersley (India) Pvt. Ltd. (2003)