Effect of a long-ranged part of potential on elastic S-matrix element

A.M.D.M. Shadini, R.A.D.Piyadasa, J.Munasinghe
Department of mathematics,University of Kelaniya, Kelaniya

ABSTRACT

It has been found that quantum mechanical three-body Schrödinger equation can be reduced to a set of coupled differential equations when the projectile can be easily breakable into two fragments when it is scattering on a heavy stable nucleus [1]. This coupled set of differential equations is solved under appropriate boundary conditions, and this method, called CDCC, has been found to be a very successful model in high energy quantum mechanical three body calculations [2]. It can be shown, however, that the coupling potentials in the coupled differential equations are actually long-range [3],[4] and asymptotic out going boundary condition, which is used to obtain elastic and breakup S-matrix elements is not mathematically justifiable. It has been found that the diagonal coupling potentials in this model takes the inverse square form at sufficiently large radial distances [3]and non-diagonal part of coupling potentials can be treated as sufficiently short-range to guarantee numeral calculations are feasible. Therefore one has to justify that the long range part of diagonal potential has a very small effect on elastic and breakup S-matrix elements to show that CDCC is mathematically sound .Although the CDCC method has been successful in many cases, recent numerical calculations[5],[6]indicate its unsatisfactory features as well. Therefore inclusion of the long range part in the calculation is also essential. The main objective of this contribution is to show that the effect of the long range part of the potentials on S-matrix elements is small.
Let is consider the Schrödinger equation related to the long-range diagonal potential in the form

$$[\frac{d^2}{dr^2}+k^2-l(l+1)-\frac{2\mu}{\hbar^2}V(r)]U_l(k,r)=0 \tag{1}$$

where $V(r)$ falls off as $\frac{1}{r^2}$ at large r . If we define $F_l(k)$ by [7]

$$F_l(k)=1+ik^l\int_0^\infty U_l(k,r)\frac{2\mu}{\hbar^2}V(r)h_l(kr)dr=F_l^{)1)}(k)+F^{(2)}_l(k) \tag{2}$$

where $h_l(kr)=j(kr)+in_l(kr)$ in the usual notation, and

$$F_l^{(1)}(k)=1-k^l\int_0^\infty U_l(k,r)\frac{2\mu}{\hbar^2}n_l(kr)dr \tag{3}$$

$$F_l^{(2)}(k)=k^l\int_0^\infty U_l(k,r)\frac{2\mu}{\hbar^2}j_l(kr)dr \tag{4}$$

It can be shown that [7] the S-matrix element $S_l(k)$ can be written as

$$S_l(k)=(-1)^l\frac{F_l^*(k)}{F_l(k)} \tag{5}$$

Now, we will show that the long range part of the potential has a minor effect on the S-matrix element. If the potential $V(r)$ take the form of inverse square form beyond R_m ,

$$F_l(k)=1+ik^l\int_0^{R_m} U_l(k,r)\frac{2\mu}{\hbar^2}V(r)h_l(kr)dr+ik^l\int_{R_m}^\infty (kr)^{1/2}\frac{2\mu\gamma}{\hbar^2 r^2}J_\nu(kr)h_l(kr)dr \tag{6}$$

Now , we will show that the term

$$F_l^{R_m}(k) = ik^l \int_{R_m}^{\infty} (kr)^{1/2} \frac{2\mu\gamma}{\hbar^2 r^2} J_\nu(kr) h_l(kr) dr \tag{7}$$

where $\nu = \eta + \frac{1}{2}$ and $\eta(\eta+1) = l(l+1) + \frac{2\mu}{\hbar^2}\gamma$ is small. If R_m is sufficiently large $h_l(kr) = e^{i(kr - \frac{1}{2}(l+1)\pi)}$, and therefore

$$F_l^{R_m}(k) = ik^l e^{-i\frac{(l+1)\pi}{2}} \int_{R_m}^{\infty} (kr)^{1/2} \frac{2\mu\gamma}{\hbar^2 r^2} J_\nu(kr) e^{ikr} dr \tag{8}$$

Due to the fact e^{ikr} is rapidly oscillating and $J_\nu(kr)$ is also oscillating taking positive and negative valves , $F_l^{R_m}(k)$ becomes small since the cancellation of many terms occur in the integration ,and the integrand decays also as $O(1/r^2)$.Hence we deduce that the long range part of the potential may have a small effect on the S-matrix element. This fact can further be justified by evaluating in closed form and is in progress.

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