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Solutions of singular Sturm-Liouville problems using exponential integrators

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Sturm-Liouville problems play a major role in the fields of applied mathematics, physics, and engineering. Therefore, the solutions of SLPs are particularly influential in these study areas. In the mid-1830s, the French mathematician Charles-François Sturm and Joseph Liouville worked independently on the problem of heat conduction through a metal bar, developing techniques for solving a large class of partial differential equations (PDEs) in the process and the study of this Sturm-Liouville problem originated. In combination with Magnus expansion, the Lie group method is used to develop an algorithm to solve the Sturm-Liouville Problem with an arbitrary set of boundary conditions. This study concerns singular Sturm-Liouville problems and their solutions. We use Magnus expansion method, which is an exponential integrator that use the exponential function of the Jacobian for the numerical integration of the problem. The coefficients of the Magnus expansion method of 6^{th}-order were derived using two-point Gaussian Quadrature rule and Lagrange interpolation method. To validate and assess the accuracy and the performance of the method, the results were compared with the analytical solution and the 4^{th}-order method for a particular singular SLP. It was observed that the absolute and relative error values given by the 6^{th}-order method are almost comparable with those of the 4^{th}-order with slight variations which can be attributed to the rounding off errors. As expected, the complexity of the 6^{th}-order method is slightly high as the floating point (FLOPS) count shows. In conclusion, the accuracy of the 4^{th} and 6^{th} order methods are the same although the complexity of 6^{th}-order method is slightly higher. In future work, it is expected to improve the 6^{th}-order method by using the properties of Lie brackets and using different quadrature rule for the integrals.

Keywords: Singular Sturm-Liouville problems, Magnus expansion, Two-point Gaussian Quadrature rule