

Analysis of a stochastic predator-prey model

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In biological systems *Lotka-Volterra* predator-prey model describes the population dynamics of two interacting species of predators and its preys. Classical predator-prey model is a primitive deterministic model governed by the two differential equations, namely,

$$dx = (b_1x - c_1xy) dt \text{ and } dy = (c_2xy - d_2y) dt$$

where x and y denote prey and predator respectively, and b_1, c_1, c_2 and d_2 are parameters.

This model can be improved by introducing stochasticity that accounts for the random fluctuations of a realistic predator-prey dynamical system. In this research work, we use Stochastic Differential Equation (SDE) approach. There are various ways, based on various assumptions, to incorporate SDE. One common approach is to use equations of the following form:

$$\begin{aligned} dx &= (b_1x - c_1xy) dt + \sqrt{(b_1 + d_1)x} dw_1 \\ dy &= (c_2xy - d_2y) dt + \sqrt{(b_2 + d_2)y} dw_2 \end{aligned}$$

These types of Stochastic Differential Equations (SDE) can be simulated in Matlab using numerical methods such as *Euler-Maruyama method*. Phase planes of the deterministic and stochastic models are carried out to demonstrate the behavior of this modified model.

Our initial goal is to compare different stochastic models with the original deterministic model through simulations. The deterministic model has a positive equilibrium which is globally stable for positive values of the parameters. Nevertheless, in the stochastic model, the predator and prey populations may tend to extinction. Extinction percentages of predator or prey population are summarized and analyzed through this research work.

Keywords: Predator-prey model, Stochastic differential equations, Matlab simulations