An Objective Approach for Prescribing Visual Acuities Through Diopter Values

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Abstract: - Many factors, including the cornea, the lens, and the irregular shape of the eyeball, might influence the visual acuity (VA) of the eye. Ocular aberrations (OAs) are induced by these imperfections, and the normalized Zernike expansion is a typical approach for describing OAs. Customarily, estimating VA of the eye has included procedures that are subjective in nature and conducted through the eye chart. Also, it has been used conversion formulas to convert lower-order aberration to diopters. However, there is no proper approach to convert the diopter values to VA. Intending to avoid the drawbacks of subjective techniques of predicting VA and for accurating the prescribing through the OAs, unlike prior work, we propose an objective approach to determine the impact of Zernike mode(s) corresponding to an ocular aberration on VA using an objective image quality (IQ) metric which is Neural Sharpness (NS). Proposed approach leads to get mathematical relationship between VA and diopters. The Summed Square of Residuals (SSE), Root Mean Square Error (RMSE), and R^2 approach confirm the accuracy and the validity of the model. This relationship can be employed to estimate the VA of any ocular aberration in terms of diopters. Also, the proposed relation can be used to assess the efficacy of refractive treatments.

Key-Words: - Diopters, logMAR, Neural Sharpness, Visual Acuity, Image Quality Metrics, Zernike Coefficients, Zernike Polynomials

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1 Introduction

Defects in the human eye reduce the retinal image quality (IQ), that is, the retinal image becomes in blurriness, distortions, and low contrast sensitivity. The ocular aberrations (OAs) can be utilized to describe the vision defects caused due to irregular shape of the eye objectively. These aberrations consist of (1) lower-order aberrations (LOAs) and (2) higher-order aberrations (HOAs) [3]. The most common ocular aberrations are lower-order aberrations such as astigmatism and refractive errors such as myopia, hypermetropia [8]. Among many high-order aberrations, spherical aberration and coma have attracted attention. The Shack-Hartmann aberrometry measure is the most commonly used measure of ocular aberrations [3]. The generalized Zernike expansion is a standard method to specify these aberrations since they have outstanding properties in comparison to other polynomials such as Fourier series and Taylor monomials [1, 3]. The properties of Zernike polynomials are described elsewhere [3]. In this case, the measured wavefront is constructed using Zernike polynomials, and expansion coefficients called Zernike coefficients which are calculated through the reconstruction.

VA is a fundamental measure of the clarity and sharpness of vision, that is, assessing the visual performance. Traditionally, VA has been assessed subjectively using standardized tests, such as Snellen charts, which rely on the observer's interpretation. There are several subjective studies that have been carried out to reveal the impact of each Zernike mode on VA [1, 9]. However, objective methods for evaluating VA have gained increasing attention in recent years, aiming to provide more accurate and reliable measurements that are less influenced by subjective factors. Nevertheless, there is no appropriate objective approach that has been conducted to determine the impact of each Zernike mode on VA. Unlike prior work, we propose an objective approach to prescribe vision defects through Zernike coefficients using a relationship between the defocus term and VA. By quantifying and understanding this relationship, researchers aim to predict visual outcomes, improve diagnostics and treatment planning, and optimize refractive surgery outcomes.

2 State of the Art

Ophthalmologists and optometrists use ocular aberrations to describe vision defects of human eyes beyond conventional refractive errors and diagnose eyes with abnormal optics [3]. The relationship between ocular aberrations and VA has been described

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and studied [1]. Applegate et al. (2003) concluded that increasing the magnitude of individual Zernike mode decreases the VA in linear manner. However, it was not discussed the relationship between individual Zernike modes and VA. The impact of individual aberrations and combinations of aberrations on VA were examined through several optical metrics [2, 6, 7]. Moreover, Ravikumar et al. (2013) studied the impact of ocular aberrations on VA and explored that certain IQ metrics such as Neural Sharpness (NS), Visual Strehl Ratio (VSX), and Light in the Bucket (LIB) are well correlated with VA for normal and keratoconus eyes. In this study, therefore we use NS as a tool to develop a relationship between diopter values and VA.

2.1 Wavefront Representation

Assume a wavefront $W(r,\theta)$ with pupil radius R. As the wavefront is continuous in $\mathbb{R}^2$, it can be written in L^2 sense as:

$$W(r,\theta) = \sum_{i=0}^{\infty} a_i Z_i(\rho,\theta), \quad (1)$$

where $\rho = r/R$ is the normalized pupil radius, a_i is the i^{th} expansion coefficient known as Zernike coefficient, and $Z_i(\rho,\theta)$ is the i^{th} Zernike polynomial (Zernike mode).

2.2 Zernike Coefficients and Spherical Equivalent

The spherical equivalent (M) is related to the defocus coefficient and can be computed [3]:

$$\text{M} = \frac{-4\sqrt{3}a_4}{R^2}, \quad (2)$$

where a_4 is Zernike coefficient for defocus and R is the pupil radius in millimeters. The value M provides the mean spherical power of the sphero-cylindrical refraction [3].

2.3 Point Spread Function

The PSF describes the response of an imaging system to a point source or point object and is analogous to the impulse response [4]. The point spread function is determined using the squared modulus of the Fourier transform of the generalized pupil function

$$\text{PSF} = |F\{P(x,y)\}|^2, \quad (3)$$

where F represents the Fourier transform operator. The generalized pupil function for the wave aberration $W(x,y)$ is:

$$P(x,y) = p(x,y)e^{i\frac{2\pi}{\lambda}W(x,y)}, \quad (4)$$

where $p(x,y)$ is the circular aperture with $p(x,y) = 1$ inside the aperture, otherwise $p(x,y) = 0$, λ is the wavelength.

2.4 Neural Sharpness

The NS is calculated by weighting the PSF with a Gaussian profile that represents the neural visual system. The product is then integrated over the domain of PSF, and normalized by the corresponding value for a diffraction-limited PSF (PSF_{DL}). In this case, the NS can be estimated:

$$\text{NS} = \frac{\iint \text{PSF}(x,y)G(x,y)dxdy}{\iint \text{PSF}_{\text{DL}}(x,y)G(x,y)dxdy}, \quad (5)$$

where $G(x,y)$ is a bivariate-Gaussian, neural weighting function [5]. Applegate et al. (2013) found that change of VA in logMAR (δ logMAR) is highly correlated with change in logNS (δ logNS) and the relationship as:

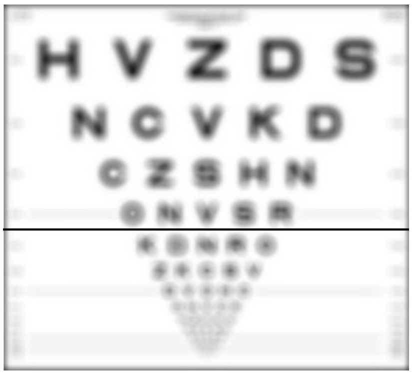
$$\delta\text{logMAR} = -0.4651\delta\text{logNS} - 0.0395. \quad (6)$$

2.5 Visual Acuity Measurement Model

2.5.1 Subjective Approach

VA is the sharpness and clarity of vision and is related to the compactness of the optical point spread function. Typically, VA can be measured subjectively by asking the patient to read down the eye chart as far as they can. Figure 1 (a) shows a typical logMAR eye chart. The calculated PSF of an abnormal eye convolves with the eye chart is shown in Figure 1 (b) and which can be utilized to conclude the VA of the eye subjectively.

(a)



(b)

Figure 1: (a) shows typical logMAR eye chart for visual acuity measurement while (b) depicts the simulation of vision outcome for an aberrated eye.

2.5.2 Objective Approach

The simplest method for computing the proper average VA from any notation is to convert the value to the logMAR equivalent and then take the average of the logMAR values. The easiest way to compute the logMAR value is to convert to the decimal notation and then take the negative of the logarithm. The formulas for going from decimal to logMAR and then back are as follows:

$$\log \text{MAR} = -\log(\text{Decimal Acuity}). \quad (7)$$

$$\text{Snellen Visual Acuity Denominator} = 20/\text{Decimal Acuity} \quad (8)$$

3 Methodology

In this section, the procedure of proposed approach for obtaining an objective relationship between diopter values and VAs has been presented in details.

3.1 Data Collection

In this study, we used a synthetic data set. The data set was generated to meet the reference range of keratoconus eyes. In particular, defocus term from 0 to certain value were partitioned with fixed step size (The SE varied from 0 D to -4 D).

3.2 Obtaining Diopter Values

The SEs for corresponding defocus values were acquired from (2). Also, equation (3) was used to calculate the PSF for each defocus value.

3.3 Obtaining Snellen Visual Acuity Values

Once, the PSF is calculated. Equation (5) yields for obtaining the logNS. Recognizing the strong correlation between δlogNS and δlogMAR, equation (6) was employed to estimate changes in VA. Following the acquisition of logMAR values, the process continued by determining Snellen decimal VA values through the application of (7). Subsequently, these decimal values underwent conversion to the Snellen VA, a critical step facilitated by the transformation outlined in (8).

3.4 Relationship between Spherical Equivalent and Visual Acuity

To establish a quantitative relationship between SEs and VA measurements, a systematic analysis was undertaken. Figure 2 illustrates the flow chart of the procedure for developing an analytical relation between diopter values and VAs.

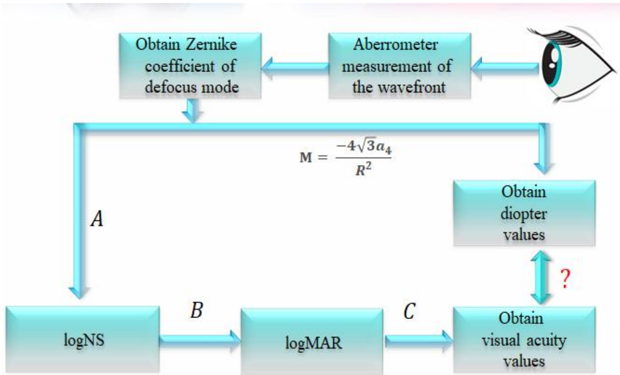


Figure 2: The methodology is summarized in the flow chart.

4 Results and Discussions

4.1 Relationship between Visual Acuity and Spherical Equivalent

The VA values were meticulously plotted against corresponding diopter values, visually represented in Figure 3.

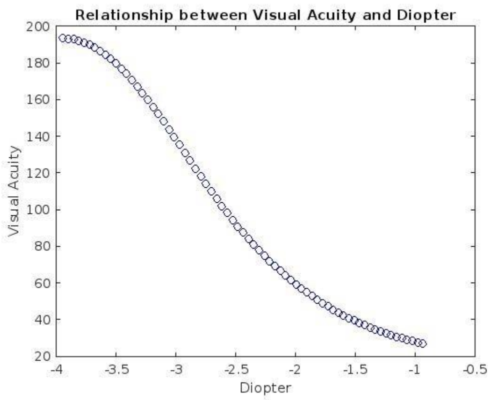


Figure 3: Distribution of the VA values verses diopters is shown.

The denominator of VA distribution against the diopters was written as Fourier and Polynomial functions as shown in (9) (Model 1) and (10) (Model 2), respectively

$$y = a_0 + \sum_{n=1}^{8}(a_n \cos(nx\omega) + b_n \sin(nx\omega)) \quad (9)$$

$$y = \sum_{i=0}^{8} c_i x^i \quad (10)$$

where x denotes diopter values and y denotes VA values and c_i denotes coefficients of polynomial function. The expansion coefficients for the Fourier approach are given in Table 1.

Table 1: Expansion Coefficient values of Fourier Function

Term	Value	Term	Value
a_0	108.0490	b_1	84.8417
a_1	8.5121	b_2	14.7482
a_2	-1.4176	b_3	6.9590
a_3	5.9575	b_4	3.8931
a_4	3.9282	b_5	1.7371
a_5	2.3232	b_6	0.7225
a_6	1.0629	b_7	0.2214
a_7	0.3916	b_8	0.0640
a_8	0.0870	ω	1.2425

The Goodness of fit statistics for each fitted curve is included in Table 2.

Table 2: The Goodness of Fit Statistics is shown.

Equation	R^2	SSE	RMSE
Model 1	≅1.0	6.00E-04	0.0034
Model 2	≅1.0	0.2073	0.0588

In comparing Model 1, which utilizes a Fourier function with Model 2, employing a polynomial function, it becomes apparent that the Fourier Model 1 offers distinct advantages over the polynomial Model 2 in this context. The perfect R-squared value of ≅ 1 for Model 1 indicates an exceptional fit, particularly beneficial when dealing with periodic data patterns that Fourier series are designed to capture efficiently. The low sum of squared errors (SSE) and root mean square error (RMSE) further reinforce the Fourier model's ability to precisely represent the underlying periodic behavior (note that close investigation shows harmonic distribution) in the dataset. On the other hand, Model 2, based on a polynomial function, demonstrates a significantly higher error, suggesting limitations in capturing the nuances of the data.

Fourier series, by their nature, are well-suited for representing periodic phenomena, providing a more appropriate framework for this particular dataset compared to polynomials. The Fourier model's success underscores the importance of selecting a modeling approach that aligns with the inherent characteristics of the data, showcasing how, in this instance, Fourier functions outperform polynomials in capturing and representing the underlying periodic trends.

In addition, the basic functions (sine and cosine waves) of a Fourier series are orthogonal, which simplifies the calculation of coefficients. This orthogonally property can make Fourier series more numerically stable and efficient for certain types of analysis. Model 2 fitting is shown by the blue line in Figure 4.

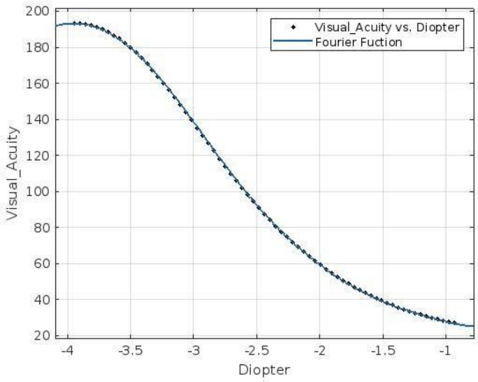


Figure 4: Model fitting for the selected Model (Model 2)

The fitted model can be used to make predictions since a good fit between actual and estimated outputs is shown by high accuracy and all errors being closer to zero. The relationship between both diopter values and 20/20 vision system measures is show in Table 3.

Table 3: Conversion of diopter values to 20/20 measure is summarized.

Diopter Value (D)	Visual Acuity (20/20 measure)
-4.00	20/193
-3.75	20/191
-3.50	20/180
-3.25	20/161
-3.00	20/138
-2.75	20/114
-2.50	20/92
-2.25	20/74
-2.00	20/59
-1.75	20/48
-1.50	20/39
-1.25	20/33
-1.00	20/28
-0.75	20/25

5 Conclusions

In this paper, a collection of Zernike coefficients corresponding to defocus were utilized to obtain a relationship between the Zernike coefficients and VA, using logNS and logMAR.

A relationship between VA and Diopter is obtained. The validity of the model was confirmed using SSE, RMSE and R^2 values. Thereby, once a patient's Zernike expansion coefficient (defocus) has been determined, it is possible to obtain the optical power of the lens that should be prescribed to eliminate a certain aberration from the eye. This model can also be used to assess the efficacy of refractive treatments. The extent to which this approach may be generalized across multiple visual targets remains to be explored.

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The authors have no conflicts of interest to declare that are relevant to the content of this article.