

Abstract No: PO-29

Easy guide to the theory of Alexandrov spaces with Curvature Bounded Below (CBB)

Malith T. A. B.* and Senanayake N. P. W. B. V. K.

Department of Mathematics, Faculty of Science, University of Kelaniya, Sri Lanka.
mr.bimsara@gmail.com*

Topological manifolds are the most general space in which many studies have been done. An Alexandrov space is also a topological manifold and is by definition a complete, and locally compact length space with Curvature Bounded Below (CBB) or Curvature Bounded Above (CBA) introduced by A.D. Alexandrov. Those spaces were discussed by A.D. Alexandrov as generalised Riemannian manifolds as long ago as 50's. Alexandrov space is still quite new in the field of differential geometry. Comparatively very few research works are available as the generalisation of the results originally published for manifolds. There remains a lack of easily accessible educational resources that provide a comprehensive and understandable guide to this theory for individuals with varying levels of mathematical background. So, the main purpose of this research is to give a simple and thorough explanation of the theory of Alexandrov spaces with CBB. Importantly, the proofs of those theorems, propositions and corollaries are provided in a detailed way and explained step by step. So, this research serves as a beginner's guide in studying the Alexandrov space. This begins with basic concepts and gradually progresses to complex ideas and proofs. For instance, we start by exploring the basic concepts such as geodesics, sectional curvature, Hausdorff topology and length spaces. Then we defined Alexandrov spaces with CBB by a constant k ($Curv(X) \geq k$). Alexandrov convexity is the basic tool we need, to define an Alexandrov space and it is a kind of space where we cannot do any calculation, to overcome that we consider a model space denoted by $M^n(k)$; The n -dimensional complete, simply connected space of constant sectional curvature k . Then we provide a detailed analysis of angles within Alexandrov spaces with CBB, supported by five lemmas that outline the properties of these angles and their implications for the structure of the space. One of the lemmas discussed a key differentiating factor between CBB and CBA, which states that any geodesics in CBB do not branch. Finally, we proved the Toponogov Comparison theorem, using the lemma on limit angles and the lemma on narrow triangles. This theorem stands out as a fundamental tool in the discussion of Alexandrov spaces. By the end of this research, readers will have a fundamental understanding of Alexandrov spaces with CBB and their significance in differential geometry making this an easily accessible educational resource and essential guide for students and researchers interested in geometry, topology and beyond.

Keywords: Alexandrov space, Curvature Bounded Below, Differential geometry, Topological manifolds, Toponogov Comparison theorem