



Section E1

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Effect of long-range part of the potential on the elastic S-matrix element

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The quantum mechanical three-body Schrödinger equation can be reduced to a set of coupled differential equations when the projectile is easily breakable into two fragments and when scattering is a heavy stable nucleus. It has been found that the diagonal coupling potentials in this model take the inverse square form at sufficiently large radial distances and non-diagonal part of coupling potentials can be treated as sufficiently short-range to guarantee that numeral calculations are feasible. We will show that this long-range part of the potential has a small contribution to the elastic S-matrix element.

Let us consider the Schrödinger equation related to the long-range diagonal potential in the form

$$\left[\frac{d^2}{dr^2} + k^2 - \frac{l(l+1)}{r^2} - \frac{2\mu}{\hbar^2} V(r) \right] U_l(k, r) = 0$$

where $V(r)$ falls off as $\frac{1}{r^2}$ at large r . If we define $F_l(k)$ by

$$F_l(k) = 1 + ik^l \int_0^\infty U_l(k, r) \frac{2\mu}{\hbar^2} V(r) h_l(kr) dr$$

where $h_l(kr) = j_l(kr) + in_l(kr)$ in terms of spherical Bessel and Neumann functions.

S-matrix element $S_l(k)$ can be written as $S_l(k) = (-1)^l \frac{F_l^*(k)}{F_l(k)}$

Now, we will show that the long-range part of the potential has a minor effect on the S-matrix element. If the potential $V(r)$ takes the form of inverse square form beyond R_m ,

$$F_l(k) = 1 + ik^l \int_0^{R_m} U_l(k, r) \frac{2\mu}{\hbar^2} V(r) h_l(kr) dr + F_l^{R_m}$$

and

$$F_l^{R_m}(k) = A_l ik^l \int_{R_m}^\infty (kr)^{1/2} \frac{2\mu\gamma}{\hbar^2 r^2} J_\nu(kr) h_l(kr) dr = A_l (-1)^{\frac{(l+1)\pi}{2}} ik^l \int_{R_m}^\infty (kr)^2 \frac{2\mu\gamma}{\hbar^2 r^2} J_\nu(kr) e^{ikr} dr$$

where $\nu = \eta + \frac{1}{2}$, $\eta(\eta+1) = l(l+1) + \frac{2\mu}{\hbar^2} \gamma$ and A_ℓ is a constant. Due to the fact e^{ikr} is rapidly oscillating and $J_\nu(kr)$ is also oscillating taking positive and negative values, $F_l^{R_m}(k)$ becomes very small since the cancellation of many terms occur in the integration, and the integrand decays also as $O(1/r^2)$. We set $R_m \approx 30 fm$ and calculated $F_l^{R_m}(k)$ and found that it is very small. Hence, we conclude that the long-range part of the potential has a very small effect on the elastic S-matrix element.