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PAPER

The velocity of a particle relative to an observer instantaneously at rest coinciding with the point through which the particle passes in a spherical distribution of matter comprising electrically counterpoised dust with constant uniform density

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A sphere comprising a special kind of matter, electrically counterpoised dust in which all the elastic forces have been cancelled out, has been considered.

A static spherically symmetric solution to Einstein's field equations has been found using a new set of boundary conditions. In introducing these new boundary conditions, we assume that the radial coordinates in and out of the sphere need not be the same and we are guided by the notion of what may be called proper distances and proper times of two observers on either side of the sphere. In these new boundary conditions we replace ordinary partial derivatives by generalized partial derivatives in curvilinear coordinates.

Then the solution takes the form

$$ds^2 = \frac{1}{\left(\theta\left(\frac{r}{l}\right)\right)^2} c^2 dt^2 - \left(\theta\left(\frac{r}{l}\right)\right)^2 \left(dr^2 + r^2 d\Omega^2\right) \qquad 0 \le r \le a$$

$$ds^2 = \frac{1}{\left(1 + \frac{A_2}{R}\right)^2} c^2 dT^2 - \left(1 + \frac{A_2}{R}\right)^2 \left(dR^2 + R^2 d\overline{\Omega}^2\right) \qquad A < R$$

where $A_2 = -\frac{a^2}{l}\theta'\left(\frac{a}{l}\right)$, $\theta\left(\frac{r}{l}\right)$ is the solution of the Lane-Emden equation $\frac{1}{x^2}\frac{d}{dx}\left(x^2 \frac{dy}{dx}\right) = -y^3$, $r = lx$, l is a constant of dimension length , a is the coordinate radius of the sphere .

In our approach $r = a$ in the matter-filled region corresponds to $R = A$ in the region without matter, outside the sphere.

The velocity of a particle relative to an observer instantaneously at rest coinciding with the point through which the particle passes has been calculated for this metric.

Using these values, a minimum value for a measure of energy with which the particle has to be projected at the center of the sphere, to reach infinity has been calculated to be $\dfrac{c}{\theta\left(\frac{a}{l}\right)+\frac{a}{l}\theta'\left(\frac{a}{l}\right)}$ where c is the velocity of the light.

A minimum value for a measure of energy with which the particle has to be projected at the center of the sphere, to reach infinity has also been calculated for metric derived using standard (Lichernowicz) boundary conditions which says that the metric coefficients and their partial derivatives are continuous across the boundary of the sphere.

It is shown that we have the same value irrespective of boundary conditions used.

Also a minimum value for a measure of energy with which the particle has to be projected at the center of the sphere, to reach the exterior region of the sphere has been calculated to be $\dfrac{c}{\theta\left(\frac{a}{l}\right)}$.

The comparison of this value with the value obtained for the metric derived using standard (Lichernowicz) boundary conditions is also done and it is shown that these two values are the same irrespective of the boundary conditions used.

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