4.9 Deflection of light with cosmological constant

J.A.N.K. Jayakody[1] ,L.N.K. de Silva[2]
[1]Department of Physics, University of Kelaniya
[2]Department of Mathematics, University of Kelaniya

ABSTRACT

A beam of light is deflected when it passes near a massive object such as the Sun or a black hole, due to the gravitational influence of the massive body. Without the cosmological constant, the total deflection angle of light is $2\delta = \frac{4m}{r_0}$ [1], where r_0 is the closest approach of the light ray from the centre of the massive body and $m = \frac{GM}{c^2}$. Here G is the gravitational constant and M is the mass of the central object. Taking into account the Schwarzschild de Sitter geometry, some authors [2] have found recently, that the cosmological constant contributes to the deflection of light. In this paper, we study the effect of the cosmological constant on the deflection of light when it passes near a massive object.

The null-geodesic equation in Schwarzschild-de-sitter metric

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2),$$

where $$f(r) \equiv 1 - \frac{2m}{r} - \frac{\Lambda r^2}{3} \text{ [3]},$$

can be written as $$c^2 l^2 - h^2 u'^2 - h^2 u^2 + 2mh^2u^3 + \frac{\Lambda h^2}{3} = 0,$$

where l and h are constants, which can be written in another form of

$$u'' + u = 3mu^2 \text{ [1]}.$$

The zeroth order solution and the first order solution of the light ray trajectory are respectively given below.

$$u_0 = \frac{1}{r_0}\cos\phi \text{ [1]}$$

$$u = \frac{1}{r_0}\cos\phi - \frac{\varepsilon}{3r_0^2}\cos^2\phi + \frac{2\varepsilon}{3r_0^2} \text{ [1]}, \quad \text{where } \varepsilon = 3m.$$

Now, assume a second order solution for the light ray trajectory in the following form.

$$u = \frac{1}{r_0}\cos\phi - \frac{\varepsilon}{3r_0^2}\cos^2\phi + \frac{2\varepsilon}{3r_0^2} + \varepsilon^2 w,$$

where $w \equiv w(r_0, \phi)$.

We find that the solution up to the second order for the light ray trajectory can be written as,

$$u = \frac{1}{r_0}\cos\phi - \frac{\varepsilon}{3r_0^2}\cos^2\phi + \frac{2\varepsilon}{3r_0^2} + \frac{r_0\sin\phi}{2}\left[\frac{5\varepsilon^2}{6r_0^4}\phi - \frac{\varepsilon^2}{12r_0^4}\sin 2\phi + \left(\frac{5\varepsilon^2}{9r_0^4} + \frac{\Lambda}{3}\right)\cot\phi\right].$$

Taking the limits as, $r \to \infty, u \to 0, \phi \to \frac{\pi}{2} + \delta, \phi \to -\frac{\pi}{2} - \delta$ we find the two asymptotes of the trajectory, which give the total deflection angle as,

$$2\delta = \frac{\left[\frac{4\varepsilon}{3r_0} - \frac{8\varepsilon^2}{9r_0^2} + \frac{2\Lambda r_0^2}{3}\right]}{\left(1 - \frac{2\varepsilon}{3r_0} + \frac{\Lambda r_0^3}{2\varepsilon}\right)}.$$

Λ is shown to be a function of ε^2, and substituting $\Lambda = \varepsilon^2 k$ in the above equation, and neglecting the terms of ε, of order higher than four, we find,

$$2\delta = \frac{4\varepsilon}{3r_0} - \frac{32\varepsilon^3}{27r_0^3} - \frac{16\varepsilon^4 k}{27} + \frac{64\varepsilon^4}{81r_0^4}.$$

Writing the above equation in terms of Λ, we obtain,

$$2\delta = \frac{4\varepsilon}{3r_0} - \frac{16\varepsilon^2}{27}\Lambda - \frac{32\varepsilon^3}{27r_0^3} + \frac{64\varepsilon^4}{81r_0^4}.$$

Substituting $\varepsilon = 3m$,

$$2\delta = \frac{4m}{r_0} - \frac{16m^2}{3}\Lambda - \frac{32m^3}{r_0^3} + \frac{64m^4}{r_0^4},$$

which is the angle of deflection in the presence of the cosmological constant, neglecting fifth and higher order terms of ε.

Keywords: deflection angle, cosmological constant, Schwarzschild de Sitter metric, light ray trajectory

References

[1] Adler, R., Bazin, M., Schiffer, M., (1965) *Introduction to General Relativity*, United States of America: McGraw-Hill, Inc.

[2] Rindler, W., Ishak, M., [astro-ph]: arXiv: 0709.2948 (2007).

[3] Ishak, M., Rindler, W., Dossett, J., Moldenhauer, J., Allison, C., [astro-ph]: arXiv: 0710.4726 (2007).