Derivation of Equations that satisfy Dirac's Equation and invariance of transformations similar to Guage Transformations under mixed numbers.

W. P.T Hansameenu, L.N.K de Silva, T.P de Silva[1]
Department of Mathematics, University of Kelaniya, Sri Lanka.
Department of Mathematics. University of Sri Jayawardhanapura, Sri Lanka[1].

ABSTRACT

Mixed number is a sum of a scalar and " vector " part. If we consider the two mixed numbers $(a+\underline{A})$ and $(b+\underline{B})$ the summation and product of the mixed numbers are defined respectively as,

$$(a+\underline{A}) \oplus (b+\underline{B}) = a+b+\underline{A}+\underline{B}$$

$$(a+\underline{A})\otimes(b+\underline{B}) = ab + a\underline{B} + \underline{A}b + \underline{A}.\underline{B} + i\underline{A}\times\underline{B} \qquad (1)$$

Dirac's Equation can be written as

$$\overline{P}\,\vec{\psi} = m_0 c \vec{\psi} \qquad (2)$$

Dirac's momentum and wave function can be defined using mixed numbers as

$$\overline{P} = \left(\frac{iE}{c}, \overline{p}\right)$$

$$\vec{\psi} = \left(\frac{i\psi_0}{c}, \overline{\psi}\right)$$

Then from (1)

$$\left(\frac{iE}{c}, \overline{p}\right) \otimes \left(\frac{i\psi_0}{c}, \overline{\psi}\right) = m_0 c\left(\frac{i\psi_0}{c}, \overline{\psi}\right) \qquad (3)$$

Equating real and imaginary parts of both sides of Eq.(3),

$$\nabla\psi_0 - \frac{\partial\overline{\psi}}{\partial t} - \frac{m_0 c^2}{\hbar}\overline{\psi} + c\,\nabla\times\overline{\psi} = 0 \qquad (4)$$

$$-\frac{\hbar}{c^2}\frac{\partial\psi_0}{\partial t} - \hbar\nabla.\overline{\psi} - m_0\psi_0 = 0 \qquad (5)$$

These solutions satisfy the Dirac's Equation.

Now consider the transformation similar to Gauge Transformations,

$$\overline{\psi'} = \overline{\psi} + \nabla A$$

$$\psi_0' = \psi_0 + \frac{\partial A}{\partial t}$$

Under these transformations Eq. (4) and (5) give,

$$\nabla \psi_0' - \frac{\partial \overline{\psi'}}{\partial t} - \frac{m_0 c^2}{\hbar}\overline{\psi'} + c\nabla \times \overline{\psi'} = 0 \qquad (6)$$

$$\frac{1}{c^2}\frac{\partial \psi_0'}{\partial t} + \nabla.\overline{\psi'} + \frac{m_0 \psi_0^1}{\hbar} = 0 \qquad (7)$$

Thus Eq .(4) and (5) are invariant under these transformations.

References

(1)W P T Hansameenu, L N K de Silva, T P de Silva, ***Matrix representation of mixed numbers and quaternions*** - 10th Annual Research Symposium, 2009, University of Kelaniya. Kelaniya, Sri Lanka.

(2) Arbab. I Arbab, *The Quaternionic Quantum Mechanics,* arXiv:1003.0075vI [physicsgen-ph], 27 Feb 2010.