Useful identities in finding a simple proof for Fermat's last theorem

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ABSTRACT

Fermat's last theorem, very famous and difficult theorem in mathematics, has been proved by Andrew Wiles and Taylor in 1995 after 358 years later the theorem was stated However, their proof is extremely difficult and lengthy. Possibility of finding s simple proof, first indicated by Fermat himself in a margin of his notes , has been still baffled and main objective of this paper is, however, to point out important identities which will certainly be useful to find a simple proof for the theorem.

We have already used [1][3] the identity

$$(x+y-z)^p - x^p - y^p + z^p \equiv \frac{4p}{2^p}(x+y)(z-x)(z-y)\sum_{i+j+k=\frac{p-3}{2}}(p-1)!\frac{(x+y)^{2i}(z-y)^{2j}(z-x)^{2k}}{(2i+1)!(2j+1)!(2k+1)!}$$

where $i+j+k \geq 0$,which has been derived by Werbrusow[1] using the multinomial theorem, to point out that Fermat's last theorem can be easily proved for all odd primes p satisfying that $2p+1$ is also a prime. In case of Fermat's last theorem,

$$z^p = y^p + x^p, (x,y) = 1 \tag{1}$$

the above identity reduces to the equation

$$(x+y-z)^p = \frac{4p}{2^p}(x+y)(z-x)(z-y)\sum_{i+j+k=\frac{p-3}{2}}(p-1)!\frac{(x+y)^{2i}(z-y)^{2j}(z-x)^{2k}}{(2i+1)!(2j+1)!(2k+1)} \tag{2}$$

and (2) can be again written as

$$(x+y-z)^p = p(x+y)(z-x)(z-y)d^p \tag{3}$$

where

$$d^p = \frac{4}{2^p}\sum_{i+j+k=\frac{p-3}{2}}(p-1)!\frac{(x+y)^{2i}(z-y)^{2j}(z-x)^{2k}}{(2i+1)!(2j+1)!(2k+1)} \tag{4}$$

Since $x-(z-y) = y-(z-x) = (x+y)-z = x+y-z$,it follows that factors common to x, y, z and $x+y-z$ are in $(z-y),(z-x)$ and $(x+y)$ respectively and $p(x+y)$, for example, is $p^{pm}u^p$ if z in (1) is divisible by p . Also, it follows from(1) that $z-y$ and $z-x$ are p^{th} powers and hence they can be written as $z-y = h^p$, $z-x = g^p$ where h, g are factors of x and y respectively. Using these relations and (3), it can be shown easily that

$$g^p + h^p + 2uhdgp^m - p^{pm-1}u^p = 0 \tag{5}$$

,and this relation can be used to show that [3] Fermat's last theorem is true for all odd primes p satisfying the relation that $2p+1$ is also a prime. If $y \equiv 0(\mathrm{mod}\, p)$, h in (5) should be replaced by $-h$ .Simple proof of Fermat's last theorem for all aforementioned odd primes ,pointed out in [3] ,is actually due to the simple identity given by Werbrusow[1] and the important relation of Germain Sophie[1] that if p is a prime and $2p+1$ is also a prime, then the Fermat equation (1) may have a only solution that $xyz \equiv 0(\mathrm{mod}\, p)$.

Werbrusow [1]has derived another useful and important identity that

$$(x+y)^n-(x^n+y^n)\equiv\sum_{i=1}^{\infty}(-1)^n\frac{n}{i}.^{n-i-1}C_{i-1}x^i y^i(x+y)^{n-2i} \qquad (6)$$

where, by convention the terms in the above sum are zero when $n-2i>0$. This identify indicates that $(x+y)^p-(x^p+y^p)\equiv 0(\text{mod } p)$ for all odd primes p .It can be shown that the relation

$$g^p-2uhgdp^m-(h^p+u^p)=0 \qquad (7)$$

where h,u,g are factors of x,y,z in (1) respectively , is true when we assume that (1) has a solution such that none of x,y,z is divisible by p . This may be useful in discarding all integer solutions x,y,z that none of them is divisible by p and related to
the Fermat equation (1). A useful identity similar to(6) can also be written as

$$x^n+y^n\equiv(x+y)^n-{}^nC_1(x+y)^{n-1}x+{}^nC_2(x+y)^{n-2}x^2-\cdots+{}^nC_{n-1}(x+y)x^{n-1}$$

(8)

in which x on the right-hand side not in the powers of $(x+y)$ can be replaced by y as well. The identity (8) follows from

$$x^n+y^n=x^n+[\{x+y)-x]^n \qquad (9)$$

This identity may also be useful in finding a simple proof for Fermat's last theorem.

References

(1)Paulo Rebenboim ,Fermat's last theorem for amateurs, Spriger-Verlag (1991)
(2)Harold M.Edwards, Fermat's last theorem, A genetic introduction to algebraic number theory , Springer-Verlag (1977)
(3) R.A.D.Piyadasa, Simple analytical proofs of Fermat's last theorem for $n=5$ and for many odd primes,CMNSEM,Vol.1 No.5,July 2010, p.127-131