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Controlled K −frames in quaternionic setting

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Quaternion is an extension of complex numbers from the two-dimensional plane to four-dimensional space and forms non-commutative division algebra. A feature of quaternion is that the multiplication of two quaternions is non-commutative, from the non-commutativity the quaternionic Hilbert spaces are defined in two ways such as right quaternionic Hilbert space (V_H^R) and left quaternionic Hilbert space (V_H^L). K −frames are more general than ordinary frames in the sense that the lower frame bound only holds for the elements in the range of K, where $K$is a bounded linear operator in V_H^L. Controlled frame is one of the newest generalizations of the frame which has been introduced to improve the numerical efficiency of interactive algorithms for inverting the frame operator. In this research, the notion of a controlled K −frame is introduced in left quaternionic Hilbert space along the lines of their real and complex counterparts and some of their properties were analysed. Let V_H^L be a left quaternionic Hilbert space, $K \in B(V_H^L)$ and $C \in GL^+(V_H^L)$, where $B(V_H^L)$is the set of all bounded linear operators and $GL^+(V_H^L)$ is the set of all positive bounded linear operators in V_H^L with bounded inverse. A sequence of family $\Phi = \{\varphi_k\}_{k\in I}$ in V_H^L is called a $C -$ controlled $K -$ frame for V_H^L if there exist constants $m, M > 0$ such that $m\|K^\dagger\varphi\|^2 \le \sum_{k\in I} \langle\varphi_k\rangle\langle\varphi\rangle \le M\|\varphi\|^2$, for all $\varphi \in V_H^L$. First, we established a result that shows that any $K -$ frame is a controlled K −frame under certain conditions. Let K and C be self -adjoint with $CK = KC$. If $\Phi = \{\varphi_k\}_{k\in I}$ is a $K -$ frame for V_H^L then $\Phi = \{\varphi_k\}_{k\in I}$ is a $C -$ controlled $K -$ frame for V_H^L. Then we derived a necessary and sufficient condition for a sequence to be a controlled $K -$ frame and we have shown that every $C -$ controlled $K -$ frame is a $C^{-1} -$ controlled $K -$ frame. Suppose that $K \in B(V_H^L)$. A sequence $\Phi = \{\varphi_k\}_{k\in I}$ is a $C -$ controlled $K -$ frame for V_H^L if and only if $R(K) \subseteq R(T_{C\Phi})$, where $R(K)$ is the range of K. Suppose that $CK = KC$. If $\Phi = \{\varphi_k\}_{k\in I}$ is a $C -$ controlled $K -$ frame for V_H^L then $\Phi = \{\varphi_k\}_{k\in I}$ is a $C^{-1} -$ controlled $K -$ frame for V_H^L. Finally, we proved that the sum of two controlled $K -$ frames remains a controlled $K -$ frame under certain conditions in left quaternionic Hilbert space. Let $CK = KC$. Suppose that $\Phi = \{\varphi_k\}_{k\in I}$ and $\Psi = \{\psi_k\}_{k\in I}$ are $C -$ controlled $K -$ frames for V_H^L with bounds m, M and m', M', respectively. If $T_\Phi {T_\Psi}^\dagger = C^{-1}KK^\dagger$, then $\{\varphi_k + \psi_k\}_{k\in I}$ is also a $C -$ controlled $K -$ frame for V_H^L.

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