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An approximate solution to Lane-Emden equation of the polytrophic index three by using Differential Transform Method

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Lane-Emden equation is a second order dimensionless non-linear ordinary differential equation which can be used to describe internal structure of a star, the thermal behaviour of a spherical cloud of gas, isothermal gas spheres etc. Self-gravitating spheres of plasma, such as stars, can also be described approximately by using these equations. Lane-Emden equation was solved by using Adomian Decomposition Method (ADM), Homotopy Analysis Method for some values of polytrophic index n. There are exact, analytical solutions for Lane-Emden equation in particular values $n = 0, 1, 5$. Since its non-linearity, the exact solutions cannot be found easily. Differential Transform Method (DTM) is an iterative method with a Taylor series solution gives good approximation in very small region. DTM can be applied for both linear and nonlinear n^{th} derivative functions. In this research, a numerical solution to Lane-Emden equation with $n = 3$ has been found by using Differential Transform Method. To increase the range of convergence of the solution, the Pade approximation has been applied. Pade approximation is a ratio of two McLaurin's expansion of the polynomials. The obtained solution for Lane-Emden equation has been compared with the solutions obtained by using the Fourth Order Runga-Kutta (RK4) method, ODE45 and Forward Euler method, which are effective and accurate methods for solving differential equations. The Einstein-Maxwell equations for a static spherical distribution of matter which is called Electrically Counterpoised Dust (ECD) under gravitational attraction and electrical repulsion can be simplified to the Lane-Emden equation when $n = 3$. It has been shown that the mass of a sphere of electrically counterpoised dust is an increasing function of its radius and it has a maximum value. Since the solution obtained gives us a physically acceptable result, it can be justified that the obtained solution using DTM is acceptable and gives better approximate solution with the form of a polynomial for linear and nonlinear differential equations.

Keywords: Differential transform method, Einstein-Maxwell equations, Lane-Emden equation, Pade approximation.