

Approximate analytical solution to the time-fractional nonlinear Schrodinger equation through the Sumudu decomposition method

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The time-fractional nonlinear Schrodinger equation has the following form:

$$iD_t^\alpha u(x,t) = -\frac{1}{2}\Delta^2 u(x,t) + V_d(x)u(x,t) + \beta_d |u(x,t)|^2 u(x,t), \quad 0 < \alpha \leq 1,$$
$$u(x,0) = u_0(x), \quad x \in \mathfrak{R}^d, t > 0,$$

where V_d is the trapping potential and β_d is a real constant. The physical model of above equation and its generalized forms arise in various areas of physics, including quantum mechanics, nonlinear optics, plasma physics and superconductivity. Exact solutions of most of the fractional nonlinear Schrodinger equations cannot be found easily. Therefore, analytical and numerical methods have been used in the literature. Some of the analytical methods for solving nonlinear problems include the Adomian decomposition method, Variational iteration method and Homotopy analysis method. In this study, we use the Sumudu decomposition method to construct the approximate analytical solutions of the time-fractional nonlinear Schrodinger equations with zero and nonzero trapping potentials. The Sumudu decomposition method is a combined form of the Sumudu transform and the Adomian decomposition method. The fractional derivatives are defined in the Caputo sense. The exact solutions of some nonlinear Schrodinger equations are given as a special case of our approximate analytical solutions. The computations show that the described method is easy to apply, and it needs smaller size of computation as compared to the aforementioned existing methods. Further, the solutions are derived in a convergent series form which shows the effectiveness of the method for solving a wide variety of nonlinear fractional differential equations.

Keywords: Adomian decomposition method, Fractional derivative, Sumudu transforms, Time-fractional Schrodinger equation