4.28 On the systematic of anomalous absorption of partial waves by nuclear optical potential

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ABSTRACT

An interesting phenomenon relating to the nuclear optical potential was discovered (Kawai M & Iseri Y,(1985)) [1] which is called the anomalous absorption of partial waves by the nuclear optical potential. They found, by extensive computer calculations, that, for a special combinations of the total angular momentum (j),angular momentum(l),energy (E) and the target nuclei(A), the elastic S-matrix elements corresponding to nucleon elastic scattering become zero. This phenomenon is universal for light ion elastic scattering on composite nuclei.[2]. It is very interesting that this phenomenon occurs for the realistic nuclear optical potential and it exhibits striking systematic in various parameter planes. For example, all nuclei which absorb a partial waves of a definite node lie along a straight in the plane$(R_C, A^{\frac{1}{3}})$ as shown in the figure ,where R_C is the closest approach and A is mass number of the target nucleus. Theoretical description of this systematic has been actually very difficult, though attempts have been made by the Kyushu group in Japan. In this contribution, we explain mathematically the most striking systematic of this phenomenon.

Explanation of the systematic

Partial wave $u_l(k,r)$ of angular momentum l and incident wave number k satisfies the Schrödinger equation

$$\frac{d^2u_l}{dr^2}+\left[k^2-\frac{l(l+1)}{r^2}-\frac{2\mu}{\hbar^2}\{V(r)+iW(r)\}\right]u_l(k,r)=0 \qquad (1)$$

,where $V(r)$ is the total real part and $W(r)$ is the total imaginary part of the optical potential. Starting from this equation , one obtains

$$\frac{d}{dr}|u_l(k,r)|^2=2\int_0^r\left[|\frac{du_l}{dr}|^2-g(r)|u_l(r)|^2\right]dr \qquad (2)$$

where $g(r)=\left[k^2-\frac{2\mu}{\hbar^2}V(r)-\frac{l(l+1)}{r^2}\right]$. If $u_l(k,r)$ is the anomalously absorbed partial wave, the corresponding S-matrix element is zero and hence in the asymptotic region $|u_l(k,r)|$ is almost constant. Therefore

$$\left[\left|\frac{du_l}{dr}\right|^2-g(r)|u_l(k,r)|^2\right]=0 \qquad (3)$$

for large r. Now, from (1) and (3), it is not difficult to obtain[3] the equation

$$\frac{1}{|u_l|^2}\frac{d}{dr}|u_l|^2=-\frac{g'(r)}{2g(r)}+\frac{W_h(r)}{g(r)|u_l|^2}\int_0^r W_h(r)|u_l|^2dr \qquad (4)$$

which is valid for large r, and has been numerically tested in case of an anomalously absorbed partial waves , where $W_h(r) = -\frac{2\mu}{\hbar^2}W(r)$. If $W(r)$ decays much more rapidly than $V(r)$ in case of a partial wave under consideration $\frac{1}{|u_l|^2}\frac{d}{dr}|u_l|^2 = -\frac{g'(r)}{2g(r)}$ and by integrating this equation with respect to r, we obtain

$$|u_l(k,r)|^2 (g(r))^{\frac{1}{2}} = C \qquad (5)$$

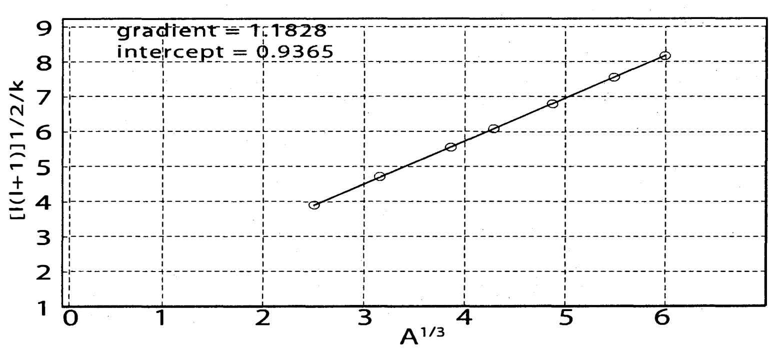
,where C is a constant, and the equation (5) is valid for large values of r. In case of anomalous absorption of the partial wave, $|u_l(k,r)|$ is constant in the asymptotic region and therefore $g(r)$ is also constant. We have found that for all partial waves corresponding to a straight line of definite node, $g(r)$ is constant at the respective closest approach. For example, at $R_C = \frac{[l(l+1)]^{\frac{1}{2}}}{k}$, $g(r)$ is constant for all partial waves lying on a straight line in case of anomalous absorption of neutron partial waves by the nuclear optical potential. Therefore, neglecting the spin-orbit potential , we get

$$\frac{2\mu}{\hbar^2}V_0[1+\exp[(\frac{[l(l+1)]^{\frac{1}{2}}}{k} - 1.17A^{\frac{1}{3}})]/a_r]^{-1} = C_0$$

where V_0 is depth of the real potential and A is the target mass and the optical potential parameter $a_r = 0.75$ and C_0 is a constant. Therefore, in case of neutron, we get the linear relation

$$\frac{[l(l+1)]^{\frac{1}{2}}}{k} = 1.17A^{\frac{1}{3}} + C_1 \qquad (6)$$

where C_1 is again a constant. This relation has found to be well satisfied in the cases we have tested numerically. The equation (6) well accounts for the anomalous absorption of neutron partial waves by the Nuclear Optical Potential as shown in the figure below.



Gradient of straight line predicted by (6) is 1.17 and the actual value is 1.1828 . Very small discrepancy is due to the negligence of the spin-orbit potential.

References

(1).Kawai M and Iseri Y , Phys. Rev.C31,400 (1985)
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(3)Piyadasa R.A.D ,Analyticity of elastic S-matrix element, to be published.