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Accelerating the rate of convergence of some efficient schemes for two-stage Gauss method

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The non-linear equations obtaining from the implicit *s* – stage *Runge-Kutta* methods have been solved by various iteration schemes. A scheme has been developed, which is computationally more efficient and avoids expensive vector transformations. The rate of convergence of this scheme is examined when it is applied to the scalar test differential equation $x' = qx$ and the convergence rate depends on the spectral radius $\rho[M(z)]$ of the iteration matrix $M(z)$, where $z = hq$ and h is the step-size. In this scheme, supremum of a lower bound for $\rho[M(z)]$ is minimized over the left half z-plane with the constraints requiring super-linear convergence at $z = 0$ and $z \to \infty$ .Two new schemes with parameters are obtained for the two-stage Gauss-method. Numerical experiments are carried out in order to evaluate and compare the efficiency of the new schemes and the original scheme.

Consider an initial value problem for stiff system of ordinary differential equations $x' = f\big(x(t)\big)$, $x(a) = v$, $f\colon \mathbb{R}^n \to \mathbb{R}^n$. An *s*-stage implicit *Runge-Kutta* method computes an approximation x_{r+1} to the solution $x\ (t_{r+1})$ at discrete point $t_{r+1} = t_r + h$ by $x_{r+1} = x_r + h\sum_{i=1}^{s} b_i\, f(y_i\,)$, where $y_{1\,,}y_{2\,,\ldots,}y_{s,}$ satisfy *sn* equations $y_i = x_r + h\ \sum_{i=1}^{s} a_{ij}\, f\big(y_j\big),), i = 1,2,\ldots,s.$ $A = \big[a_{ij}\big]$ is the real coefficient matrix and $b = [b_{1\,,}b_{2\,,\ldots,}b_s]^T$ is the column vector of the *Runge-Kutta* method.

Let $y = y_1 \oplus y_2 \oplus \ldots \oplus y_s \in \mathbb{R}^{sn}$and $F(Y) = f(y_1) \oplus f(y_2) \oplus \ldots \oplus f(y_s) \in \mathbb{R}^{sn}$. Then the above equation in $y_{1\,,}y_{2\,,\ldots,}y_s$ may be written by $Y = e \otimes x_r + h(A \otimes I_n)F(Y)$, where $e = (1,1,\ldots,1)^T$ and $(A \otimes I_n)$ is the tensor product of the matrix A with $n \times n$ identity matrix I_n. The efficient scheme, which has been already proposed, is given by

$$[I_s \otimes (I_n - h\lambda J)]E^m = (L \otimes I_n)(e \otimes X_r - Y^m) + (U \otimes I_n)(\ e\ \otimes X_r - Y^{m-1}) + h(T \otimes I_n)F(Y^m) + h(R \otimes I_n)F(Y^{m-1}),\ m = 1,2,\ldots,$$

In this scheme, supremum of a lower bound for $\rho[M(z)]$ is minimized over $\mathbb{C}^-$, where $\mathbb{C}^- = \{z \in C\ /Re\ (z) \le 0\ \}$ with the constraints $\rho[M(z)] = 0$ at $z = 0$ and $\rho[M(z)] = 0$ at $z \to \infty$. The parameters for the two-stage Gauss method are obtained and Numerical experiments are carried out.

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