Natural Frequencies of Equilateral Triangular Plates under Classical Edge Supports

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***Abstract*—In this paper, free vibration of thick equilateral triangular plates subject to classical boundary conditions has been investigated based on a new shear deformation theory, which needs no shear correction factor for stress distributions. The numerical modeling is performed by means of Rayleigh-Ritz method to obtain the concerned eigenvalue problem. The objective is to find the effect of different physical and geometric parameters on natural frequencies of the plate. New results for natural frequencies along with 3D mode shapes have been evaluated after the test of convergence and validation with the available results.**

Keywords; Vibration; Equilateral plate; Rayleigh-Ritz method, 3D mode shapes.

I. INTRODUCTION

Design engineers and architects often need plates with different geometries for convenient structural design and performance. As such, it is also worth to study the dynamical and structural behavior of triangular plates.

Gorman [1-3] has proposed a highly accurate analytical solutions (method of superposition) for free vibration of right triangular plates with simply supported edge supports, with combinations of clamped-simply supported boundary supports and different boundary conditions with one edge free respectively. Kim and Dickinson [4, 5] have studied free vibration of isotropic and orthotropic right triangular plates and of generally triangular plates respectively by using Rayleigh-Ritz method. Transverse vibrations of triangular plates have also been investigated by Singh and Chakraverty [6] with various types of boundary conditions at the edges by using boundary characteristics orthogonal polynomials as basic functions in the Rayleigh-Ritz method.

In view of the mentioned, present authors have taken a first time attempt to solve free vibration problem of thick isotropic equilateral triangular plate based on a new power-law exponent shear deformation plate theory (PESDPT). The prime objective is to evaluate the effect of various physical and geometric parameters on non-dimensional frequencies of this plate. As such, free vibration behavior is found along with 3D mode shapes after a valid test of convergence and comparison with available results.

II. PLATE THEORY

Let us consider an equilateral triangular plate which can be determined by three numbers a, b and c respectively in Cartesian coordinate system and moderate thickness of *h*. One can easily represent the displacement fields of thick isotropic triangular plate based on generalized higher-order shear deformation plate theory as below:

$$u_x = u(x,y;t) - z\frac{\partial w}{\partial x} + f(z)\phi_x(x,y;t)$$
$$u_y = v(x,y;t) - z\frac{\partial w}{\partial y} + f(z)\phi_y(x,y;t)$$
$$u_z = w(x,y;t) \tag{1}$$

where u, v, w, ϕ_x and ϕ_y are the unknown displacement components for the deformed triangular thick plate; *f(z)* is the shape function which determines the parabolic distribution of the transverse shear strains and stresses across the thickness. Present investigation assumes a new power-law exponent shear deformation plate theory (PESDPT) and accordingly $f(z) = h\left(\frac{z}{h}\right)^{2n+1} - (2n+1)z\left(\frac{1}{2}\right)^{2n}$ and this form of plate theory is the first of its kind to implement in finding natural frequencies of the thick triangular plate. Non-zero linear strains associated with this theory can now be expressed as:

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + f(z)\frac{\partial \phi_x}{\partial x} \quad (2a)$$

$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y} = \frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + f(z)\frac{\partial \phi_y}{\partial y} \quad (2b)$$

$$\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - 2z\frac{\partial^2 W}{\partial x \partial y} + f(z)\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right) \quad (2c)$$

$$\gamma_{xz} = \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = f'\phi_x \quad (2d)$$

$$\gamma_{yz} = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} = f'\phi_y \quad (2e)$$

Based on these strains of Eq. (2), one may write the linear stresses in the following matrix form:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{66} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (3)$$

where the reduced stiffness components are $Q_{11} = Q_{22} = \frac{E(z)}{1-\nu^2}$, $Q_{12} = Q_{21} = \frac{\nu E(z)}{1-\nu^2}$ and $Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}$.

III. MECHANICAL ENERGIES

Using Eqs. (2) and (3), the strain (U) and kinetic (T) energies concerned with the deformed plate can be defined as follows by eliminating certain redundant terms in original expressions.

$$\begin{aligned} U = \int_\Omega \Bigg[& A_{11}\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right\} + 2A_{12}\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} \\ & + A_{66}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + D_{11}\left\{\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\right\} \\ & + 2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \\ & + E_{11}\left\{\left(\frac{\partial \phi_x}{\partial x}\right)^2 + \left(\frac{\partial \phi_y}{\partial y}\right)^2\right\} + 2E_{12}\frac{\partial \phi_x}{\partial x}\frac{\partial \phi_y}{\partial y} \\ & + E_{66}\left(\frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial y}\right)^2 - 2F_{11}\left(\frac{\partial \phi_x}{\partial x}\frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi_y}{\partial y}\frac{\partial^2 w}{\partial y^2}\right) \\ & - 2F_{12}\left(\frac{\partial \phi_x}{\partial x}\frac{\partial^2 w}{\partial y^2} + \frac{\partial \phi_y}{\partial y}\frac{\partial^2 w}{\partial x^2}\right) \\ & - 4F_{66}\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)\frac{\partial^2 w}{\partial x \partial y} + H_{44}\left(\phi_x^2 + \phi_y^2\right)\Bigg] dxdy \end{aligned} \quad (4)$$

and

$$\begin{aligned} T = \int_\Omega \Bigg[& \rho_0\left\{\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right\} + \rho_2\left\{\left(\frac{\partial^2 w}{\partial x \partial t}\right)^2 \right. \\ & \left. + \left(\frac{\partial^2 w}{\partial y \partial t}\right)^2\right\} + \rho_0^2\left\{\left(\frac{\partial \phi_x}{\partial t}\right)^2 + \left(\frac{\partial \phi_y}{\partial t}\right)^2\right\} \\ & - 2\rho_1^1\left\{\frac{\partial \phi_x}{\partial t}\frac{\partial^2 w}{\partial x \partial t} + \frac{\partial \phi_y}{\partial t}\frac{\partial^2 w}{\partial y \partial t}\right\}\Bigg] dxdy \end{aligned} \quad (5)$$

The stiffness coefficients of Eq. (4) and cross-sectional inertial coefficients of Eq. (5) are well-established for higher-order shear deformation theory and their expressions can be found in earlier works frequently. Now the displacement components can be expressed as harmonic type as below:

$$\begin{aligned} & u(x,y;t) = U(x,y)\exp(i\omega t), v(x,y;t) = V(x,y)\exp(i\omega t) \\ & w(x,y;t) = W(x,y)\exp(i\omega t), \phi_x(x,y;t) = \frac{1}{a}\Phi_x(x,y)\exp(i\omega t) \\ & \phi_y(x,y;t) = \frac{1}{c}\Phi_y(x,y)\exp(i\omega t) \end{aligned} \quad (6)$$

Here, U, V, W, Φ_x, Φ_y are the amplitudes of the displacement components and ω is the natural frequency. Substituting these forms of displacement in strain and kinetic energies, we obtain the maximum strain and kinetic energies of the following forms.

$$U_{\max} = \int_{\Omega}\left[A_{11}\left\{\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2\right\} + 2A_{12}\frac{\partial U}{\partial x}\frac{\partial V}{\partial y}\right.$$
$$+ A_{66}\left(\frac{\partial V}{\partial x} + \frac{\partial U}{\partial y}\right)^2 + D_{11}\left\{\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2\right\}$$
$$+ 2D_{12}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4D_{66}\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2$$
$$+ \frac{E_{11}}{a^4}\left\{\left(\frac{\partial \Phi_x}{\partial x}\right)^2 + \frac{1}{\mu^2}\left(\frac{\partial \Phi_y}{\partial y}\right)^2\right\} + \frac{2E_{12}}{ac}\frac{\partial \Phi_x}{\partial x}\frac{\partial \Phi_y}{\partial y}$$
$$+ \frac{E_{66}}{a^2}\left(\frac{\partial \Phi_x}{\partial x} + \frac{1}{\mu}\frac{\partial \Phi_y}{\partial y}\right)^2 - \frac{2F_{11}}{a}\left(\frac{\partial \Phi_x}{\partial x}\frac{\partial^2 W}{\partial x^2}\right.$$
$$\left. + \frac{1}{\mu}\frac{\partial \Phi_y}{\partial y}\frac{\partial^2 W}{\partial y^2}\right) - \frac{2F_{12}}{a}\left(\frac{\partial \Phi_x}{\partial x}\frac{\partial^2 W}{\partial y^2} + \frac{1}{\mu}\frac{\partial \Phi_y}{\partial y}\frac{\partial^2 W}{\partial x^2}\right)$$
$$\left. - \frac{4F_{66}}{a}\left(\frac{\partial \Phi_x}{\partial y} + \frac{1}{\mu}\frac{\partial \Phi_y}{\partial x}\right)\frac{\partial^2 W}{\partial x \partial y} + \frac{H_{44}}{a^2}\left(\Phi_x^{\,2} + \frac{\Phi_y^{\,2}}{\mu^2}\right)\right] dxdy$$

(7)

and

$$T_{\max} = \int_{\Omega}\left[\rho_0\left(U^2 + V^2 + W^2\right) + \rho_1\left\{\left(\frac{\partial W}{\partial x}\right)^2\right.\right.$$
$$\left. + \left(\frac{\partial W}{\partial y}\right)^2\right\} + \frac{\rho_0^2}{a^2}\left(\Phi_x^{\,2} + \frac{\Phi_y^{\,2}}{\mu^2}\right)$$
$$\left. - \frac{2\rho_1^1}{a^2}\left\{\Phi_x\frac{\partial W}{\partial x} + \frac{1}{\mu}\Phi_y\frac{\partial W}{\partial y}\right\}\right] dxdy$$

(8)

In general, the triangle of Cartesian coordinate system is to be mapped into right-angled triangle of natural coordinate system and the transformation involves

$$\xi = \frac{1}{a}\left(x - \frac{by}{c}\right); \eta = \frac{y}{c} \text{ and}$$

$$x = a\xi + b\eta; y = c\eta$$

and the maximum strain and kinetic energies take the following expressions.

$$U_{\max} = \frac{Dac}{2a^4}\int_{\Omega}\left[12\delta^2\left\{\left(\frac{\partial U}{\partial \xi}\right)^2 + \left(-\theta\frac{\partial V}{\partial \xi} + \frac{1}{\mu}\frac{\partial V}{\partial \eta}\right)^2\right.\right.$$
$$+ 2\nu\frac{\partial U}{\partial \xi}\left(-\theta\frac{\partial V}{\partial \xi} + \frac{1}{\mu}\frac{\partial V}{\partial \eta}\right) + \left(\frac{1-\nu}{2}\right)$$
$$\left.\left(-\theta\frac{\partial U}{\partial \xi} + \frac{1}{\mu}\frac{\partial V}{\partial \eta} + \frac{\partial V}{\partial \xi}\right)\right\} + \left\{\left(\frac{\partial^2 W}{\partial \xi^2}\right)^2 + \right.$$
$$\left(\theta^2\frac{\partial^2 W}{\partial \xi^2} - \frac{2\theta}{\mu}\frac{\partial^2 W}{\partial \xi \partial \eta} + \frac{1}{\mu^2}\frac{\partial^2 W}{\partial \eta^2}\right)^2 + 2\nu\left(\frac{\partial^2 W}{\partial \xi^2}\right)$$
$$\left(\theta^2\frac{\partial^2 W}{\partial \xi^2} - \frac{2\theta}{\mu}\frac{\partial^2 W}{\partial \xi \partial \eta} + \frac{1}{\mu^2}\frac{\partial^2 W}{\partial \eta^2}\right) + 2(1-\nu)$$
$$\left.\left(-\theta\frac{\partial^2 W}{\partial \xi^2} + \frac{1}{\mu}\frac{\partial^2 W}{\partial \xi \partial \eta}\right)^2\right\} + 12C_1\left\{\left(\frac{\partial \Phi_x}{\partial \xi}\right)^2 + \right.$$
$$\left(-\theta\frac{\partial \Phi_y}{\partial \xi} + \frac{1}{\mu}\frac{\partial \Phi_y}{\partial \eta}\right)^2 + \frac{2\nu}{\mu}\frac{\partial \Phi_x}{\partial \xi}\left(-\theta\frac{\partial \Phi_y}{\partial \xi} + \frac{1}{\mu}\frac{\partial \Phi_y}{\partial \eta}\right)$$
$$\left. + \left(\frac{1-\nu}{2}\right)\left[-\theta\frac{\partial \Phi_x}{\partial \xi} + \frac{1}{\mu}\left(\frac{\partial \Phi_x}{\partial \eta} + \frac{\partial \Phi_y}{\partial \xi}\right)^2\right]\right\}$$
$$- 24C_2\left\{\frac{\partial \Phi_x}{\partial \xi}\frac{\partial^2 W}{\partial \xi^2} + \frac{1}{\mu}\left(-\theta\frac{\partial \Phi_y}{\partial \xi} + \frac{1}{\mu}\frac{\partial \Phi_y}{\partial \eta}\right)\right.$$
$$\left(\theta^2\frac{\partial^2 W}{\partial \xi^2} - \frac{2\theta}{\mu}\frac{\partial^2 W}{\partial \xi \partial \eta} + \frac{1}{\mu^2}\frac{\partial^2 W}{\partial \eta^2}\right)$$
$$+ 2\nu\left[\frac{\partial \Phi_x}{\partial \xi}\left(\theta^2\frac{\partial^2 W}{\partial \xi^2} - \frac{2\theta}{\mu}\frac{\partial^2 W}{\partial \xi \partial \eta} + \frac{1}{\mu^2}\frac{\partial^2 W}{\partial \eta^2}\right)\right.$$
$$\left. + \frac{1}{\mu}\left(-\theta\frac{\partial \Phi_y}{\partial \xi} + \frac{1}{\mu}\frac{\partial \Phi_y}{\partial \eta}\right)\frac{\partial^2 W}{\partial \xi^2}\right] + 2(1-\nu)$$
$$\left.\left(-\theta\frac{\partial \Phi_x}{\partial \xi} + \frac{1}{\mu}\left(\frac{\partial \Phi_x}{\partial \eta} + \frac{\partial \Phi_y}{\partial \xi}\right)\right)\left(-\theta\frac{\partial^2 W}{\partial \xi^2} + \frac{1}{\mu}\frac{\partial^2 W}{\partial \xi \partial \eta}\right)\right\}$$
$$\left. + 6C_3(1-\nu)\delta^2\left(\Phi_x^{\,2} + \frac{\Phi_y^{\,2}}{\mu^2}\right)\right] d\xi d\eta$$

(9)

and

$$T_{\max} = \frac{\rho h \omega^2 ac}{2}\int_{\Omega}\left[\left(U^2 + V^2 + W^2\right) + \frac{1}{12\delta^2}\left\{\left(\frac{\partial W}{\partial \xi}\right)^2\right.\right.$$
$$\left. + \left(-\theta\frac{\partial W}{\partial \xi} + \frac{1}{\mu}\frac{\partial W}{\partial \eta}\right)^2\right\} + \frac{C_1}{\delta^2}\left(\Phi_x^{\,2} + \frac{\Phi_y^{\,2}}{\mu^2}\right)$$
$$\left. - \frac{2C_2}{\delta^2}\left\{\Phi_x\frac{\partial W}{\partial \xi} + \frac{1}{\mu}\left(-\theta\frac{\partial W}{\partial \xi} + \frac{1}{\mu}\frac{\partial W}{\partial \eta}\right)\right\}\right] d\xi d\eta$$

(10)

Here, $\theta = \frac{b}{c}$, $\mu = \frac{c}{a}$ and $\delta = \frac{a}{h}$.

IV. RAYLEIGH-RITZ APPROXIMATION

In this numerical approximation, the amplitude of respective displacement components involved in Eqs. (9) and (10) are to be expressed as linear combination of simple algebraic polynomials generated from Pascal's triangle.

$$U = \sum_{i=1}^{n_p} c_i \varphi_i^u, V = \sum_{j=1}^{n_p} d_j \varphi_j^v, W = \sum_{k=1}^{n_p} e_k \varphi_k^w$$

$$\Phi_x = \sum_{l=1}^{n_p} g_l \varphi_l^1, \Phi_y = \sum_{m=1}^{n_p} h_m \varphi_m^2$$

In these series, c_i, d_j, e_k, g_l and h_m are the unknown constants to be determined and the admissible functions φ_i^u, φ_j^v, φ_k^w, φ_l^1 and φ_m^2 must satisfy the essential boundary conditions and may be denoted as

$$\varphi_i^u(x,y) = a_f \psi_i^u(x,y), \varphi_j^v(x,y) = a_f \psi_j^v(x,y)$$
$$\varphi_k^w(x,y) = a_f \psi_k^w(x,y), \varphi_l^1(x,y) = a_f \psi_l^1(x,y)$$
$$\varphi_m^2(x,y) = a_f \psi_m^2(x,y) \tag{11}$$

where $a_f = \xi^p \eta^q (1-\xi-\eta)^r$ depends on the geometry of the plate and the indices $i, j, k, l, m = 1(1)n_p$. The parameter p takes the value 0, 1 or 2 according to as the side $\xi = 0$ is free (F), simply supported (S) or clamped (C). Similar interpretations can be given to the parameters q and r corresponding to the sides $\eta = 0$ and $\xi + \eta = 1$ respectively. Consequently one can find the generalized eigenvalue problem for every kind of triangular plates by equating the maximum strain and kinetic energies to find the Rayleigh quotient and deriving partially with respect to unknown constant coefficients, but we have considered here only equilateral triangular plate and the concerned geometric parameters are: $\theta = 1/\sqrt{3}$ and $\mu = \sqrt{3}/2$ as given in Fig. 1.

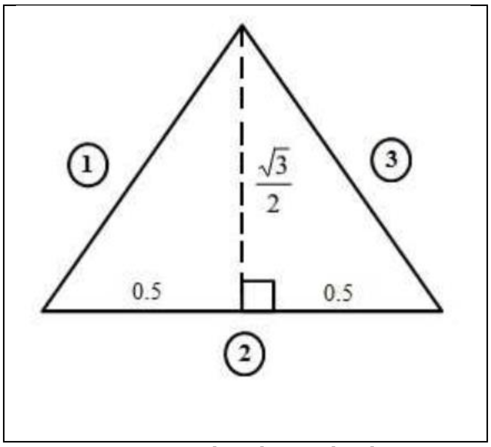

Figure 1. Equilateral triangular plate

V. NUMERICAL RESULTS

In this section, the non-dimensional frequencies for the thick equilateral triangular plate have been evaluated for different parameters after the test of convergence and comparison with the available results in special case for C-C-C edge condition with $\delta = 100$ in TABLE I. It can easily be found that the eigenfrequencies are converging with increase in number of polynomials involved in displacement components and present results are found to be in good agreement with [6].

In addition, TABLE II shows the effect of base-to-thickness ratios on non-dimensional frequencies of the concerned plate and it reveals that frequencies follow descending pattern with an increase in such ratios irrespective of the boundary conditions assumed. In addition, Fig. 2 represents first four lowest mode shapes of the concerned plate with C-C-F edge support.

TABLE I. CONVERGENCE AND COMPARISON OF FIRST FIVE NATURAL FREQUENCIES OF C-C-C PLATE WITH $\delta = 100$

n_p	MODES				
	1	**2**	**3**	**4**	**5**
10	99.0296	190.8624	190.8624	327.6569	344.2996
15	99.0034	189.2425	189.2425	302.1998	319.9431
18	99.0000	188.9388	189.1213	299.6874	317.5551
21	98.9954	188.9377	188.9377	296.5598	316.4923
[6]	99.022	189.05	189.22	296.85	316.83

TABLE II. EFFECT OF δ ON FIRST FIVE NATURAL FREQUENCIES OF EQUILATERAL PLATES

1-2-3	δ	**1**	**2**	**3**	**4**	**5**
C-C-C	100	98.9954	188.938	188.9377	296.5598	316.4923
	20	98.3614	186.276	186.2758	289.7738	308.6506
	10	96.4534	178.615	178.6145	199.9884	199.9884
	5	89.7729	99.9942	99.9942	124.4727	155.2155
C-C-S	100	81.5817	165.029	165.2579	267.8886	286.5050
	20	81.0817	162.807	163.0272	261.9261	279.6384
	10	79.5750	156.392	156.5896	198.4968	198.7460
	5	74.2760	99.2484	99.3730	123.8073	136.6496
C-C-F	100	40.0155	95.8337	101.7871	173.6736	195.3130
	20	39.8686	94.9668	100.6223	170.8116	191.2387
	10	39.4183	92.3906	97.2071	122.0249	145.8796
	5	37.7409	61.0124	72.9398	83.7407	86.2455
S-S-S	100	52.6272	122.848	122.8481	218.0862	235.5702
	20	52.3525	121.368	121.3677	213.5650	230.2998
	10	51.5209	117.065	117.0649	194.5083	194.5083
	5	48.5514	97.2541	97.2541	103.5285	103.5285

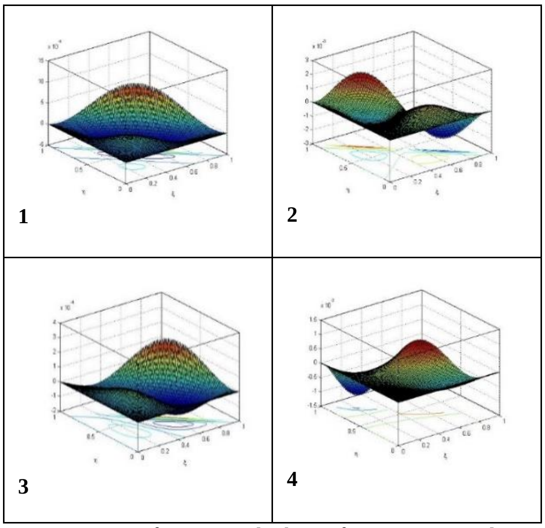

Figure 2. First four 3D mode shapes for C-C-F triangular plate

VI. CONCLUSIONS

In this investigation, natural frequencies of the isotropic thick equilateral triangular plate have been evaluated by means of Rayleigh-Ritz approximation. On this note, we may summarize the following results:

- In Rayleigh-Ritz method, the number of polynomials plays a crucial role in finding free vibration of triangular plate.
- Natural frequencies go on converging with an increase in a number of polynomials in Rayleigh-Ritz method regardless of edge conditions.
- Triangular base-to-thickness ratios have major importance in getting natural frequencies, which follow descending pattern with an increase in such ratios.
- Like PESDPT, we may also find alternate forms of plate theories for different mechanical problems.

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B. Appendix

$$C_1 = \int_{-0.5}^{0.5} \left\{\bar{z}^{2n+1} - (2n+1)\bar{z}(0.5)^{2n}\right\}^2 d\bar{z}$$

$$C_2 = \int_{-0.5}^{0.5} \left\{\bar{z}^{2n+2} - (2n+1)\bar{z}^2(0.5)^{2n}\right\} d\bar{z}$$

$$C_3 = \int_{-0.5}^{0.5} \left[(2n+1)^2 \left\{\bar{z}^{2n} - (0.5)^{2n}\right\}^2\right] d\bar{z}$$

where $\frac{-1}{2} \leq \bar{z}\left(= \frac{z}{h}\right) \leq \frac{1}{2}$.

REFERENCES

[1] D.J. Gorman, "A highly accurate analytical solution for free vibration analysis of simply supported right triangular plates," Journal of Sound and Vibration, vol. 89(1), pp. 107–118, 1983.

[2] D.J. Gorman, "Free vibration analysis of right triangular plates with combinations of clamped–simply supported boundary conditions," Journal of Sound and Vibration, vol. 106(3), pp. 419–431, 1986.

[3] D.J. Gorman, "Accurate free vibration analysis of right triangular plate with one free edge," Journal of Sound and Vibration, vol. 131(1), pp. 115–125, 1989.

[4] C.S. Kim, and S.M. Dickinson, "The free flexural vibration of right triangular isotropic and orthotropic plates," Journal of Sound and Vibration, vol. 141(2), pp. 291–311, 1990.

[5] C.S. Kim, and S.M. Dickinson, "The free flexural vibration of isotropic and orthotropic general triangular shaped plates." Journal of Sound and Vibration, vol. 152(3), pp. 383–403, 1992.

[6] Singh, and S. Chakraverty, "Transverse vibration of triangular plates using characteristic orthogonal polynomials in two variables," Int. J. Mech. Sci., vol. 34(12), pp. 947–955, 1992.