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Maximising the total population in a diffusive predator-prey system using optimal control theory

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Studying population dynamics provides insights into the behavioural patterns of species. Several scholars have introduced different physical systems to understand behavioural patterns of real-world phenomena. Predator-prey interaction is one of the most common interrelationships in nature. Studying a diffusive predator-prey model is significant for several reasons which represents a more realistic representation of population dispersal in space, where a heterogeneous population exists. In this study, optimal control theory has been applied to a diffusive predator-prey system in analysing the population model to achieve the maximum total population. By implementing a time-dependent control variable $\alpha(t)$ which is the mixing rate, to the diffusive predator-prey system, our concern is to maximise both predator and prey populations in a final time T. According to optimal control theory, under mathematical formulation, the Pay-off functional and the Hamiltonian are introduced along with the adjoint dynamics and the terminal conditions where the optimality criterion is satisfied. The Pontryagin Maximum Principle has been introduced such that the Hamiltonian is maximised where $0 \leq \alpha(t) \leq 1$. More interestingly, we determined that the Hamiltonian would get its maximum value when $p_2\mu_2 - p_1\mu_1 \leq 0$, where the optimal value of the control variable is $\alpha^*(t) = 0$ and if $p_2\mu_2 - p_1\mu_1 > 0$ then $\alpha^*(t) = 1$, where p_1, p_2 are costate functional and μ_1, μ_2 are the rate of prey loss and the growth rate of predators respectively. Furthermore, the term $p_2\mu_2 - p_1\mu_1$ is analysed under three cases which are $\mu_2 < \mu_1, \mu_2 = \mu_1$ and $\mu_2 > \mu_1$. From the analytical approach, we obtained that for the cases $\mu_2 < \mu_1$ and $\mu_2 = \mu_1$ the system is uncontrollable, and a switching time does not exist. The study reveals that the system is considered controllable only when $\mu_2 > \mu_1$ where a switching time exists such that the total population is maximised. A simulation is conducted under the case $\mu_2 > \mu_1$ to depict the results, achieving the maximum population at $t = 16.5$ while setting the switching time as $t = 11$. Finally, the obtained results were validated. Therefore, using both numerical simulation and estimates we concluded that $\alpha^*(t) = 0$ when $[0, t_2]$ and $\alpha^*(t) = 1$ when $(t_2, T]$, where t_2 is the switching time. In future work, it is expected to improve the analysis by implementing both time and space dependent control variable $\alpha(x, t)$.

Keywords: Diffusive predator-prey system, Maximum population, Optimal control theory, Pontryagin Maximum Principle, Switching time