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Optimal choice of the shape parameter for the radial basis functions method in one-dimensional parabolic inverse problems

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Inverse problems have many important applications in science, engineering, medicine, and various other fields. Numerous analytical and numerical methods exist for solving these problems. In the present work, we obtain the numerical solution of a one-dimensional inverse parabolic equation with energy overspecification at a point using the Radial Basis Functions (RBF) method. The RBF method is a meshless approach based on the collocation method, which does not require evaluating any integrals. The collocation matrix is severely ill-conditioned, and the performance and accuracy of the method highly depend on the choice of the radial basis function and the shape parameter. We examine the performance and effectiveness of three different RBFs: Gaussian, multiquadric, and inverse multiquadric, with regard to the solutions of the inverse problem. To validate our approach, we compare our results with known analytical solutions for two test problems. Furthermore, we calculate the optimal shape parameter for the three radial basis functions for the one-dimensional parabolic inverse problem with energy overspecification. It was concluded that, in general, the multiquadric RBF (MQRBF) and inverse multiquadric RBF (IMQRBF) are better options than the Gaussian RBF (GRBF) in terms of error in both the inverse and direct problems. We observed that the error in the approximation of the source control function decreases linearly with time and increases with the step size, a behavior consistent across all three RBFs considered in the study. From the analysis carried out in search of the optimum shape parameter, we found that the optimal values of the shape parameters for the GRBF, MQRBF, and IMQRBF are 1, 2, and 2, respectively, considering both error norms. Furthermore, we have observed that IMQRBF is better suited for the current inverse problem due to the smaller condition number of the collocation matrix.

Keywords: Energy overspecification, Optimal shape parameter, Parabolic inverse problem, Radial basis functions method