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Equipment Replacement: A Stochastic Approach Considering Financial and Failure Costs

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There are various motives to substitute productive equipment, with equipment degradation being one of the main factors. Equipment degradation involves the expenses of malfunctions that become costly over time, resulting in loss to the company. However, there is a scarcity of research that approaches the problem from a stochastic perspective. The incorporation of randomness in decision models signifies a progression in the quality of equipment replacement decisions. Thus, this research aims to analyze the advantages of the stochastic approach compared to the deterministic one, which is widely adopted. The results indicate that it is not advisable to generalize procedures that use a single sample size of failure data as a rule for equipment replacement decisions.

Keywords: *Equipment Replacement, Failure Costs, Financial Costs, Stochastic Financial Model*

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Introduction

Deciding when to replace equipment is a common problem for many companies, it is complex, as it involves many factors that are difficult to measure, and generally uses financial methods, such as NPV (Net Present Value) and EAUC (Equivalent Annual Uniform Costs). In operations management, deciding when to equipment replacement is a critical decision, as it involves large financial amounts and the decision is difficult to reverse (Hirschfeld, 2000; Rösiö & Bruch, 2017; Koschnick & Hartman, 2020). According to Degarmo (1973) and Yatsenko and Hritonenko (2022), a fundamental concept in this context is that the economic life of equipment corresponds to the time interval, generally in years, in which the EAUC of owning and operating the equipment asset reaches its minimum value.

There are several reasons to replace production equipment, one of the factors in particular is the deterioration of the equipment, which involves operational costs. In this regard, Hartman and Tan (2014) assess that when deciding to replace equipment, the operating and maintenance costs of current equipment and advances in equipment technology available on the market must be considered. Edokpia and Oparah (2013) also show the importance of maintenance in increasing reliability, and how it affects the decision of the equipment replacement.

Tambe and Kulkarni (2016) state that the factors used in the decision of the equipment replacement must be considered in an integrated manner and that the fact that the performance of equipment in production, when improved by the introduction of preventive maintenance, in excess results can be considered as unnecessary costs.

On the other hand, if the equipment is not well maintained, it can affect product quality, resulting in a large amount of rework and scrap to further increase non-quality costs and operational costs. Therefore, there is a close relationship between the maintenance of current equipment and the replacement of new equipment, as carrying out maintenance over time becomes increasingly frequent and costly until the decision to replace it is made (Seidgar et al., 2015; Bensoussan et al., 2022).

Regarding equipment reliability and how this factor influences the replacement decision, Nodem (2011) presents an interesting perspective when investigating the integration of preventive and corrective maintenance policy with replacement for a machine subject to random failures. The intensity of failures and repairs increases with the greater number of failures. Nodem (2011) introduced a reduction factor into its model to reduce equipment repair times, if preventive maintenance is carried out before failures. The author modeled this problem using the system as a semi-Markov process that allowed the history of stochastic failures to be taken into account, thus adding a new dimension to the repair/replacement theory. The author shows that implementing this type of preventive maintenance increases the useful life of the machine and that the results obtained are particularly useful for industrial systems that suffer losses due to increased repair time.

Therefore, it is observed that the introduction of randomness in decision models represents an advance in the quality of decisions about equipment replacement. There are several who approach the subject in a deterministic way, such as Yoo and Sung (1985) who constructed an indicator to determine the replacement age of equipment, and Baskakova et al., (2008), who analyzed the influence of repair and replacement costs of production equipment through a mathematical model that considers the minimum and maximum cost limits established for different types of repairs. For the special case of geometric stochastic costs, the real options approach was extended to cost-minimizing asset replacement in Yatsenko and Hritonenko (2017). Kirstein and Visser (2017) develop risk modeling for heavy mobile equipment, aiming to determine optimal replacement ages, considering the randomness of failure. Bensoussan et al. (2022) study equipment maintenance and capital investment strategy, considering random equipment failures through a stochastic method originating from an extension of Dynamic Programming. Yatsenko and Hritonenko (2022) developed stochastic algorithms based on real options and dynamic programming for medical imaging equipment using the EAUC method, but without considering the issue of variables related to failure and quality costs.

Nevertheless, Yeung et al. (2008) find that few studies integrate equipment quality and performance into the equipment replacement decision. Hartman and Tan (2014), and Bensoussan et al. (2022) also emphasize that although the topic has been studied for a long time, there is little guidance provided in the literature for cases in which cost information is insufficient, as well as a lack of useful guidance on the appropriate way to carry out replacement or optimization of the decision to replace. Another problem highlighted by the authors is that there is a lack of work that approaches the problem in a stochastic way, with deterministic models predominating. Yatsenko and Hritonenko (2022) further reinforce that equipment replacement methods with stochastic costs dependent on time and age remain an open theoretical question.

To contribute to minimizing the gaps presented previously, this article aims to propose a stochastic model for equipment replacement and analyze the advantages of the stochastic approach to the deterministic approach, which is widely used. For this analysis, a comparison was made between the approach of a deterministic method of minimum cost curves based on EAUC and the approach of a stochastic method of minimum cost curves based on EAUC, with theoretical data.

The article is divided into the following sections, in addition to this introduction: section 2 provides a summary of the theoretical framework, which supports the proposal; in section 3 the scientific procedure used is summarized; in section 4 the deterministic simulation is presented; in section 5, a stochastic approach is developed in the decision to replace equipment based on failure costs; and in section 6 the conclusion of the study is presented.

Literature Review

Characterization of the problem of replacement equipment

Equipment replacement emerged with the work of Taylor (1923) and Hotelling (1925), who analyzed it from the perspective of equipment depreciation. Since then, several policies on equipment replacement have been proposed, such as Alchian (1952) who adopted the present value of cost as a decision method. Hartman and Tan (2014) propose three methods to solve the equipment replacement problem, the EAUC, Dynamic Programming, and extension of dynamic programming with a stochastic approach.

Thus, the method is EAUC, it assumes that technology does not change over an infinite horizon of time, and can also be referred to as stationary cost, in the sense that an asset is replaced with the purchase of a new, identical one, with the same cost (Grant et al., 1990; FAN et al., 2012; Hartman; Tan, 2014; Yatsenko & Hritonenko, 2022). The EAUC is given by:

$$\mathrm{EAUC}_{(n)} = \left(\frac{(r(1+r)^{n})}{(1+r)^{n}-1}\right)\left(p + \left(\frac{s_n}{(1+r)^{n}}\right) + \sum_{i=1}^{n} \frac{o_i}{(1+r)^{i}}\right) \tag{1}$$

Where, EAUC (n) represents the equivalent annual cost of equipment with a service life of N periods, r is the interest rate per period, p is the purchase price of a new equipment, s_n is the redemption value of an equipment of n-period age, and o_i is the cost of operation and maintenance throughout period i. Therefore, the economic life of an asset is represented by n, which results in the minimum limit of the EAUC (Fan *et al.*, 2012; Hartman; Tan, 2014; Yatsenko; Hritonenko, 2022).

The Dynamic Programming method is used to solve the problem of replacement technology. The first study on the topic was pioneered by Bellman (1957), in which the author introduced the concept of Dynamic Programming. Subsequently, a significant body of research has employed this approach (Hartman, 2004; Hartman & Murphy, 2006; Yatsenko & Hritonenko, 2017).

Subsequently, the stochastic method was developed from an extension of dynamic programming. This method is approached in a stochastic manner, considering that according to Koowattanatianchai and Charles (2015), Yatsenko and Hritonenko (2017), and Bensoussan et al. (2022), the traditional and deterministic approach, assumes that there is no uncertainty about the quantity of all sources of revenue and costs associated with the equipment. However, in stochastic processes, the revenues and costs of assets are assumed to move with the equipment. Still, optimal substitution decisions in stochastic problems depend on the decision maker's risk profile, which could be different for each company (Hartman & Tan, 2014; Yatsenko & Hritonenko, 2022).

Koowattanatianchai and Charles (2015), and Yatsenko and Hritonenko (2017), also consider that several model parameters, such as costs and technological processes, are uncertain in the real-world scenario and can invalidate the ideal solution with a deterministic approach. However, the solution to this problem often involves probability estimates, which, in situations where there is limited data, produce biased analyzes (Hartman; Tan, 2014). Therefore, the closed solution to the stochastic substitution problem is not available, as solving the dynamic formulation in such situations requires simulation techniques, which can be complex and are certainly difficult to apply (Lohmann, 1986; Yatsenko & Hritonenko, 2022).

Nevertheless, according to Hartman and Tan (2014), Yatsenko and Hritonenko (2017), Bensoussan et al. (2022), and Yatsenko and Hritonenko (2022), further research is still needed that addresses forecast horizons capable of offering guidance regarding the amount of cost information needed. Furthermore, it is crucial to explore the interaction between replacement and utilization decisions, which requires different configurations of interactions between the decision-making process and traditional operational models.

Integration between the cost of non-quality and equipment replacement decision

Non-quality costs can be classified as inevitable and avoidable costs. The unavoidable costs are failure prevention costs and assessment costs, aimed at verifying compliance with specifications. Avoidable costs are considered internal failure costs and external failure costs. This "thinking" about avoidable costs of non-quality reinforces the importance of quality management for improving productivity by reducing operational costs (Tambe & Kulkarni, 2016; Duan et al., 2020). Regarding this, Crosby (1980), and Bensoussan et al. (2022), state that reducing the costs of non-quality is an opportunity to increase profit, and it is important to use this factor to understand and communicate the economic-financial value.

Therefore, it is known that the decision-making process to replace equipment includes failures that cannot be repaired or failures that can be repaired with major maintenance (Kirstein & Visser, 2017). To justify the variable that implies maintenance in a production environment, the Life Cycle Costing (LCC) analysis concept is introduced, involving reliability and maintenance.

Life Cycle Costing (LCC)

Life Cycle Costing (LCC) is a method that involves concepts about maintenance and economic aspects. It can be defined as a method for calculating the total cost of a system, from inception to disposal. The calculation of the total cost is the sum of costs associated with all stages comprising the equipment's lifecycle, including purchase price, installation cost, operating costs, maintenance, upgrade costs, and the residual value or resale value at the end of ownership or its useful life (Artemenkov et al., 2023; Cheng et al., 2023; Mostafaei et al., 2023). To evaluate the LCC, four economic methods are used: NPV; the Benefit-Cost Ratio (BCR); the Internal Rate of Return (IRR), and the Payback Period (Elyamany & El-Nashar, 2016; Lotfi et al., 2023).

According to Gluch and Baumann (2004), AlJaber et al. (2023) Kokare et al. (2023), and Mecheter et al. (2023), among the costs related to LCC, acquisition costs, maintenance and operation costs, and ownership costs stand out. Acquisition costs are related to the investment required to acquire the equipment, while maintenance and operating costs cover all costs involved in maintenance activities throughout the equipment's life cycle and all costs for the equipment to keep running. This category includes costs to man hours, parts for maintenance and operation, materials, production downtime for maintenance, energy, administration, security, etc. Ownership costs, on the other hand, encompass all costs except for those related to operation and can be subdivided into three categories: acquisition, maintenance, and sale costs.

Method

Methodological Approach

Conceptually, the quantitative approach applied through modeling and simulation of a sample collected using computational techniques, had the purpose of simulating the functioning of a production system based on mathematical models, as defined by Chen et al. (2020). In this article, the effects of the Mean Time Between Failures (MTBF), used in LCC methods, were simulated and its financial effects were analyzed. The decision analysis will be carried out through the statistical distribution of different scenarios, considering that the input variables behave according to a given probability distribution. In this case, we opted to use the negative exponential distribution for MTBF, that is, the number of failures in a given period occurs according to a Poisson distribution. This is the fact that modeling and simulation research aims to solve a real problem based on its structure and the behavior of its variables (Bertrand & Fransoo, 2002). What differentiates this article from classic proposals based on the mean is the use of extreme values such as the minimum equipment cost to seek an analysis closer to the reality of the industries.

Therefore, this study uses the quantitative approach and the simulation research method, in which the analyzed data comes from a theoretical illustrative case, referring to Ebeling (1997), section 17.6, on page 424.

Simulation

Simulation has the advantage of controlling input variables and manipulating them simultaneously. To achieve this, the dependent variable of the proposed stochastic model was the EAUC of the current equipment and the new equipment. For the development of the proposed model, the simulation was carried out with the Maple 13 software, using the stochastic numerical approach.

The variables that were observed and analyzed in the proposed stochastic model relating to the equipment are:

- Equipment age. Yoo and Sung (1985), Cassady et al. (1998), Tan and Hartman (2010), Koowattanatianchai and Charles (2015), Seidgar et al, (2015), Hartman (2004), Yatsenko and Hritonenko (2017), and Yatsenko and Hritonenko (2022) state that to analyze the replacement it is necessary to consider this variable, given that age is an important factor in the deterioration of equipment;

- Repair cost. Cassady et al. (1998), Edokpia and Oparah (2013), Felice and Petrillo (2013), Tambe and Kulkarni (2016), and Yatsenko and Hritonenko (2017) consider these costs relevant because excessive repair costs can adversely affect profitability until it becomes unviable;
- Failure rate (Mean Time Between Failures). Yoo and Sung (1985), Edokpia and Oparah (2013), Tambe and Kulkarni, (2016), and Bensoussan et al. (2022), assert that these costs are also significant as they impact the quality of the production system;
- Production per hour (production time). Hart and Cook (1995), Hartman (2004), Tan and Hartman (2010), Tambe and Kulkarni (2016), and Rösiö and Bruch (2017) state that it is essential to consider long-term productivity goals in equipment replacement decisions;
- Machine price (acquisition cost). Hart and Cook (1995), Tan and Hartman (2010), and Rösiö and Bruch (2017) consider this variable important for the financial feasibility of replacement;
- Internal failure cost (unit cost of scrap and rework). Yoo and Sung (1985), Tambe and Kulkarni (2016), and Bensoussan et al. (2022), emphasize the importance of considering failure costs as they impact equipment productivity.

Findings of the Study

Deterministic Approach

Based on the deterministic model proposed by Assis (2013), in which the Operational Expenditures (CapEx) and Capital Expenditures (OpEx) of the defender (current equipment) and the challenger (new or candidate equipment) were estimated by calculating the remaining life of the current equipment and life cycle of new equipment. The estimate used by the author was the extrapolation of the number of failures from the history recorded in the registry. In this type of estimate, the predicted number of failures each year is multiplied by the mean cost of a failure (material + labour) and preventive maintenance costs are added by the adopted policy.

To apply the model, the author used theoretical data from Ebeling's (1997), which assumes that a welding robot has been operating for 5 years, and its estimated remaining lifespan is another 5 years. This robot, referred to as a defender, currently experiences a high failure rate. Each failure results in downtime, with a mean cost of €550 (opportunity costs due to lost production and repair costs, including materials and labor). Its current market value is €12,000, and its depreciation from this value is estimated at 20% per year.

A new robot (challenger) currently costs €40,000 and the depreciation of this value is also estimated at 20% per year. Its useful life is estimated at 10 years. The robot's mean working regimen is 8 hours per day and 240 days per year, and it is expected to maintain this schedule in the coming years. The robot's registration shows the occurrence of 30 failures since its acquisition until today (time of occurrence of the last failure). The accumulated operating times until each failure are found in Table 1. For a real update rate of 15% per year, one wants to know when it will be economically viable to replace the robot based on MTBF.

Table 1

Cumulative Times Until Each Failure (hours)

1,339	1,857	2,307	3,329	3,792
3,891	5,541	5,646	5,726	5,806
6,530	6,736	6,771	6,826	7,056
7,065	7,097	7,771	7,779	7,942
8,045	8,088	8,558	8,642	8,764
8,958	9,034	9,104	9,318	9,523

Source: Ebeling (1997).

To determine the economic life of the challenger and the timing of replacement of the defender, the uniform annual costs of the two alternatives "Maintain or replace the current robot" for each of the sub-alternatives are calculated: "Postpone the replacement for another 1, 2, 3, *i* years up to 5, in the case of the defender robot and 1, 2, 3, *i* years up to 10, in the case of the challenger robot.

Considering the time period represented by *i* in years, the equipment's *Market value* in Euros (*€*) to *i* years, the equipment's working hours represented by *t* in *i* years, *m(t)* representing the equipment's working hours in *i* years, *m(ti-ti-1)* representing the current period *i* minus the previous period, maintenance cost in Euros symbolized by *MC* (*€*), maintenance cost in Euros updated for the *i* periods represented by *MC updated* (*€*), and the EAUC for the *i* period in Euros symbolized by *EAUC* (*€/year*). Subsequently, the sub-alternatives are compared and conclusions are drawn. Thus, Table 2 shows the EAUC of the challenger robot, and Table 3 shows the EAUC values of the defender robot.

Table 2

EAUC of the Challenger Robot

Years (i)	**Market value (€)**	**t (hours)**	**$m(t)$**	**$m(t_i-t_{i-1})$**	**MC (€)**	**MC updated (€)**	**EAUC (€/ano)**
1	32,00	1,920	0.88285	1	550	478	14,550
2	25,60	3,840	4.06135	3	1,65	1,248	13,759
3	20,48	5,760	9.91685	6	3,30	2,170	13,328
4	16,38	7,680	18.68329	9	4,95	2,830	13,085
5	13,10	9,600	30.53666	12	6,60	3,281	12,974
6	10,48	11,520	45.62008	15	8,25	3,567	12,958
7	8,389	13,440	64.05512	18	9,90	3,722	13,014
8	6,711	15,360	85.94795	22	12,1	3,956	13,161
9	5,369	17,280	111.39322	25	13,7	3,909	13,336
10	4,295	19,200	140.47650	29	15,9	3,943	13,557

Source: Adapted from Assis (2013)

Table 3

EAUC of the Defender Robot

Years (i)	**Market value (€)**	**t (hours)**	**$m(t)$**	**$m(t_i - t_{i-1})$**	**MC (€)**	**MC updated (€)**	**EAUC (€/ano)**
1	9,600	11520	45.62008834	15	8,25	7,174	12,450
2	7,680	13440	64.05511691	18	9,90	7,486	12,827
3	6,144	15360	85.94795129	22	12,1	7,956	13,392
4	4,915	17280	111.3932233	25	13,7	7,862	13,894
5	3,932	19200	140.476505	29	15,9	7,930	14,454

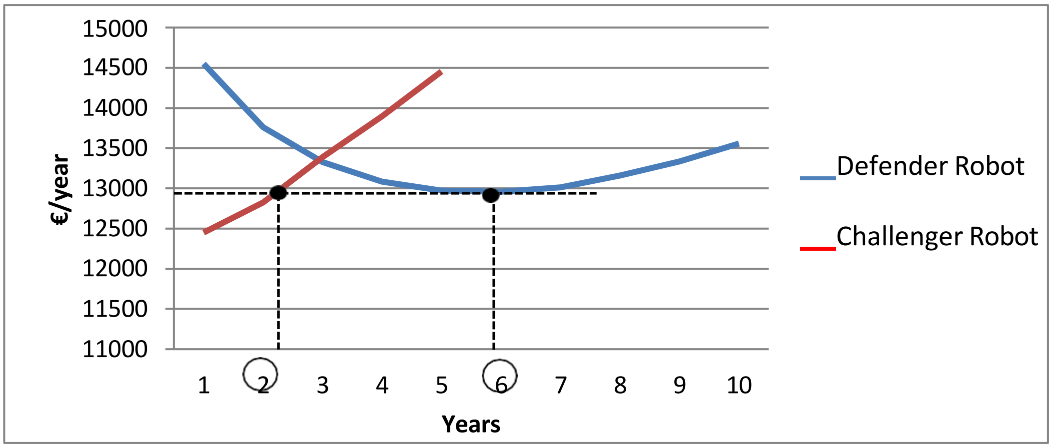
Source: Adapted from Assis (2013)

In Table 2, it is observed that the EAUC will be minimal in the sixth year (economic life) at €12,958/year. The same procedure is applied to the defender robot, resulting in Table 3. The graphical representation of the EAUC for various alternatives is presented in Figure 1.

The analysis of the results was based on the minimum cost of the challenger equipment compared to the cost of the defender robot, showing that it will be more cost-effective to keep the defender robot for an additional 2 years and replace it in the third year (i.e., at the intersection point of the two cost curves). In that year, its EUAC was €12,827/year, lower than the minimum EAUC provided by the challenger robot at €12,958/year. After this acquisition, the equipment should be retained for 6 years and replaced in the sixth year. Furthermore, this calculation should be repeated whenever one or more cost factors or market value change.

Figure 1

Variation of EUAC with robot age



Source: Adapted from Assis (2013).

Results of the Stochastic Simulation Analysis

To develop the proposed stochastic model, a numerical stochastic simulation approach was applied to the theoretical data from Ebeling (1997), assuming that the Mean Time Between Failures (MTBF) follows an Exponential distribution based on the times between failures, as shown in Table 4. In the proposed model, the times between failures are not independent, meaning the failure rate represented by $\rho(t)$ varies with time *(t)*, where *m(t)* represents the equipment's working hours in *i* years, considering $m(t) = a.t^b$, and *ln(t)* represents the inverse of the Exponential function for each time *t* found.

However, by assuming that the MTBF distribution is exponential, it is assumed that the failure rate is constant for each period and increases between periods (*i*), meaning that the MTBF decreases over time. However, the deterministic model being analyzed is based on an analysis of the equipment replacement moment that relies on a mathematical expectation, in other words, on a mean result of the minimum cost.

Table 4

Calculation of the failure intensity function and MTBF at each failure moment

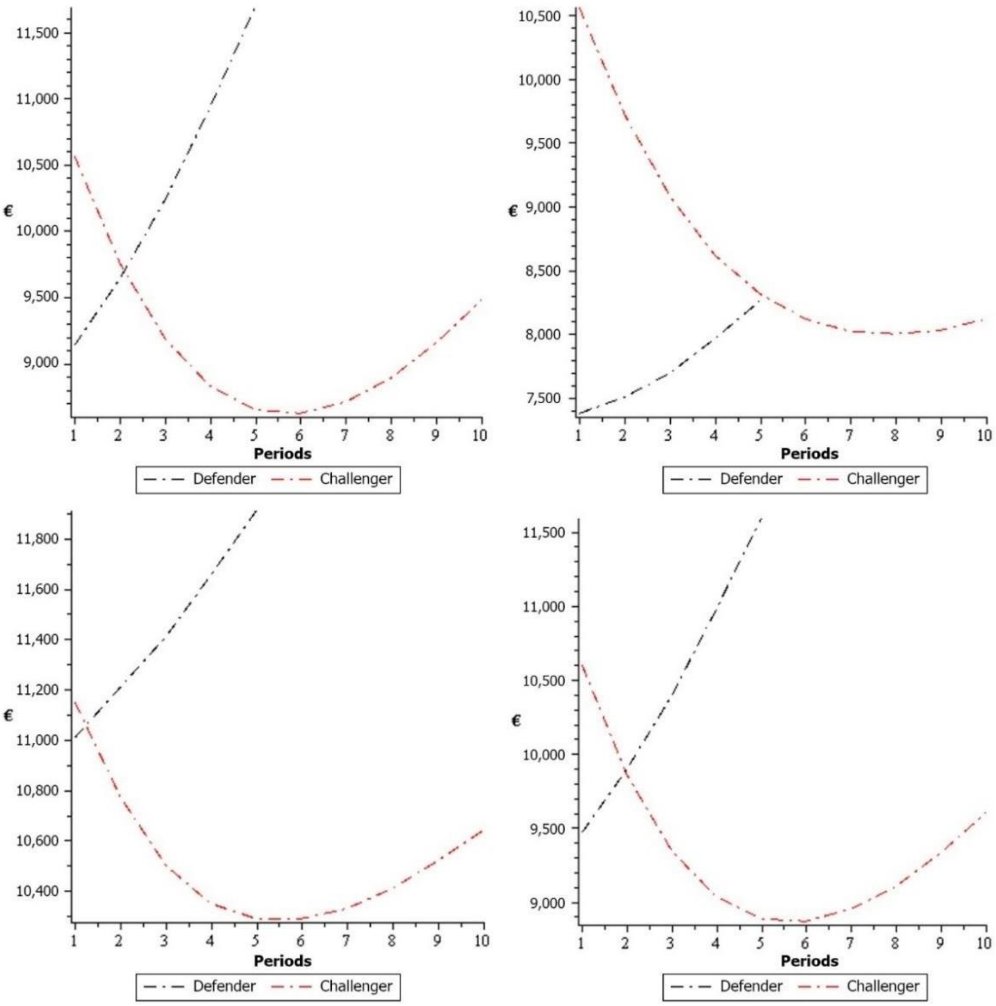
Períodos (*i*)	***t***	**ln(*t*)**	***m*(*t*)**	***r* (*t*)**	**MTBF**
1	1,390	7.199678	0.399278	0.000657	1,523.15468
2	1,857	7.526718	0.820329	0.000973	1,028.16526
3	2,307	7.743703	1.322720	0.001262	792.17028
4	3,329	8.110427	2.965697	0.001961	509.83080
5	3,792	8.240649	3.950424	0.002294	435.97710
6	3,810	8.266421	4.181068	0.002366	422.68127
7	5,541	8.619930	9.105623	0.003618	276.38693
8	5,660	8.638703	9.489856	0.003701	270.22173
9	5,726	8.652772	9.788433	0.003764	265.69123
...	...	...	...	...	...
...	...	...	...	...	...
...	...	...	...	...	...
26	8,980	9.100302	26.220304	0.006444	155.17165
27	9,034	9.108751	26.712582	0.006510	153.60426
28	9,104	9.116469	27.170422	0.006571	152.18607
29	9,318	9.139703	28.596487	0.006757	147.99569
30	9,523	9.161465	30.000000	0.006936	144.17553

In the proposed stochastic model, the distribution of minimum and maximum costs of both the challenger and defender equipment was considered together, as depicted in Figure 2, represented by graphs related to these costs. Thus, based on the MTBF sample from Table 4, it is observed that in Figure 2, the intersection points of the cost curves varied in each graph.

It is also noticeable in Figure 2 that the generated graphs considered the random variation of MTBF. Consequently, the fixed failure rate for each period, as per an exponential model with parameter $\gamma = \frac{1}{MTBF}$, results in varying equilibrium points between the costs of the defender and challenger equipment. However, in the deterministic model, the cost equilibrium point occurred in the third period (3), considering only a single sample and a failure rate that varies from equipment to equipment. Thus, the deterministic model's result is static and has limitations that require improvements that can be solved through the stochastic method.

Figure 2

Stochastic behavior of minimum costs



For this purpose, 1,000 simulations of the failure rate for similar equipment were conducted, and the equilibrium points were determined using different sample sizes: $n = [1, 2, 5, 10]$. Therefore, Table 5 shows the frequency of equilibrium points occurring in periods $T = [1, 2, 3, 4, 5]$.

Table 5

Frequency distribution of the equilibrium points for a simulation of 1000 cases

n	T	Frequency	Cumulative (F)	%	Cumulative (%)
1	1	439	439	43.90	43.90
	2	59	498	5.90	49.80
	3	55	553	5.50	55.30
	4	47	600	4.70	60.00
	5	400	1,000	40.00	100.00
2	1	551	551	55.10	55.10
	2	91	642	9.10	64.20
	3	65	707	6.50	70.70
	4	58	765	5.80	76.50
	5	235	1,000	23.50	100.00
5	1	716	716	71.60	71.60
	2	106	822	10.60	82.20
	3	53	875	5.30	87.50
	4	35	910	3.50	91.00
	5	90	1,000	9.00	100.00
10	1	845	845	84.50	84.50
	2	82	927	8.20	92.70
	3	41	968	4.10	96.80
	4	13	981	1.30	98.10
	5	19	1,000	1.90	100.00

Consequently, it is observed that the distribution of equilibrium points is asymmetrical, being truncated for a sample size of one (*n=1*). It is also noteworthy that the mean of the 1,000 simulations conducted for *n* sample sizes is close to the equilibrium point found in the deterministic model, represented by *T=3*. The simulation results indicate that the best decision is to replace the equipment in the first period, after the fifth year of using the defender equipment, which differs from the recommendation of the deterministic method.

A Box Plot was also constructed after 1,000 simulations of the results for the equilibrium points, as illustrated in Figure 3 and Table 6.

Figure 3

Box plot of the stochastic behavior of the equilibrium point for different sample sizes in a thousand simulations

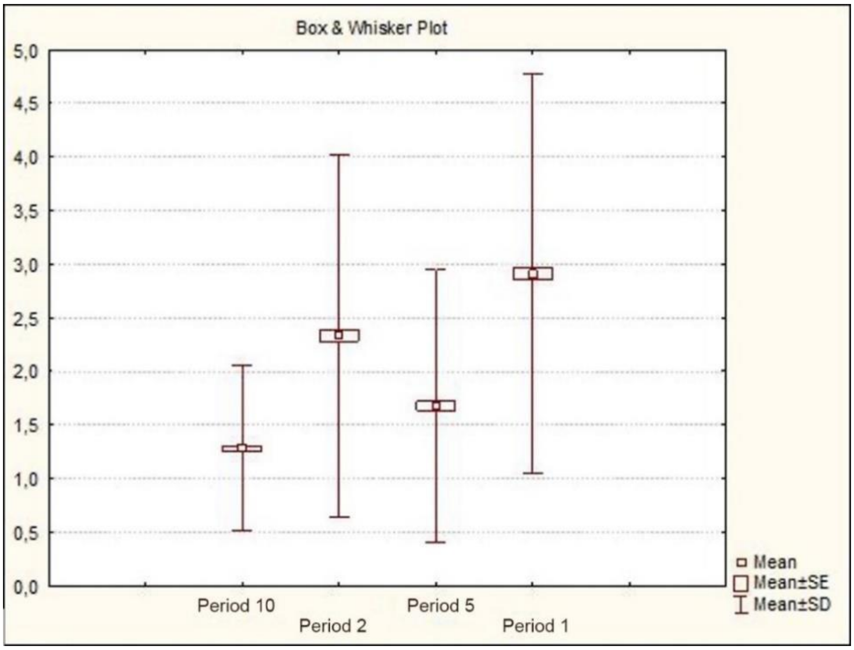


Table 6

Mean and standard deviation of thousand simulations with different sample sizes

Period	Mean	Std.Dev	Minimum	Maximum
10	1.28	0.77	1.00	5.00
2	2.34	1.69	1.00	5.00
5	1.68	1.27	1.00	5.00
1	2.91	1.86	1.00	5.00

In Figure 3 and Table 6, it can be observed that for *n=1*, referred to as Period 1, the mean equilibrium point is close to 3, which is significantly higher than the mean obtained for *n=10*, indicated as Period 10. Based on the results obtained, it is not advisable to generalize the deterministic method's procedure, as it considers only a single sample size as a decision rule. Therefore, the proposed stochastic model indicated that the best alternative is to replace the equipment in Period 1, rather than in Period 2 as suggested by the deterministic model.

Discussion and Recommendation

The use of a stochastic approach enables decision-makers to consider random variations when determining the appropriate moment to replace equipment. The results of the proposed model indicated that the equilibrium point between the cost of the challenger equipment and the defender equipment occurs in the first period in 84.5% of cases.

The deterministic model analyzes machine by machine, meaning the sample size is *n=1*, using only a single sample. On the other hand, the proposed stochastic model suggests incorporating all possible information about equipment failure rates into historical data, thereby increasing the sample size. This not only assists in more accurate decision-making but also allows for scenario analysis, considering equipment improvements.

The proposed stochastic model contributes to decision-making related to equipment replacement by considering the entire machine's life cycle and optimizing its utilization until the end of its productive life. Thus, the incorporation of life cycle considerations and the application of stochastic methods perform a crucial role in promoting sustainability, both from a financial and environmental perspective (Mecheter et al., 2023). This supports avoiding premature disposal of equipment with remaining useful life, contributing to more responsible and effective resource management.

In conclusion, the results indicate that introducing a stochastic approach to equipment replacement decisions simultaneously increases data availability and optimizes the decision-making process. It is also concluded that it is not advisable to generalize a procedure that relies on a single sample size as a decision rule.

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