Effect of long-range part of the potential on the elastic S-matrix element

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The quantum mechanical three-body Schrödinger equation can be reduced to a set of coupled differential equations when the projectile is easily breakable into two fragments and when scattering is a heavy stable nucleus. It has been found that the diagonal coupling potentials in this model take the inverse square form at sufficiently large radial distances and non-diagonal part of coupling potentials can be treated as sufficiently short-range to guarantee that numerical calculations are feasible. We will show that this long-range part of the potential has a small contribution to the elastic S-matrix element.

Let us consider the Schrödinger equation related to the long-range diagonal potential in the form

\[ \frac{d^2}{dr^2} + k^2 - \frac{l(l+1)}{r^2} - \frac{2\mu}{\hbar^2} V(r)U_i(k, r) = 0 \]

where \( V(r) \) falls off as \( \frac{1}{r^2} \) at large \( r \). If we define \( F_i(k) \) by

\[ F_i(k) = 1 + ik \int_0^\infty U_i(k, r) \frac{2\mu}{\hbar^2} V(r)h_i(kr)dr \]

where \( h_i(kr) = j(kr) + in_i(kr) \) in terms of spherical Bessel and Neumann functions.

S-matrix element \( S_i(k) \) can be written as \( S_i(k) = (-1)^i \frac{F_i^*(k)}{F_i(k)} \)

Now, we will show that the long-range part of the potential has a minor effect on the S-matrix element. If the potential \( V(r) \) takes the form of inverse square form beyond \( R_m \),

\[ F_i(k) = 1 + ik \int_0^\infty U_i(k, r) \frac{2\mu}{\hbar^2} V(r)h_i(kr)dr + F_i^{R_m} \]

and

\[ F_i^{R_m}(k) = A_i ik \int_{R_m}^{\infty} (kr)^{1/2} \frac{2\mu r}{\hbar^2} J_\nu(kr)h_i(kr)dr = A_i (-1)^i \frac{(i+1)^{1/2}}{2} \int_{R_m}^{\infty} (kr)^{1/2} \frac{2\mu r}{\hbar^2} J_\nu(kr)e^{\eta r}dr \]

where \( \nu = \eta + \frac{1}{2}, \eta(l+1) = l(l + 1) + \frac{2\mu}{\hbar^2} \) and \( A_i \) is a constant. Due to the fact \( e^{\eta r} \) is rapidly oscillating and \( J_\nu(kr) \) is also oscillating taking positive and negative values, \( F_i^{R_m}(k) \) becomes very small since the cancellation of many terms occur in the integration, and the integrand decays also as \( O(1/r^2) \). We set \( R_m = 30 fm \) and calculated \( F_i^{R_m}(k) \) and found that it is very small. Hence, we conclude that the long-range part of the potential has a very small effect on the elastic S-matrix element.