



502/E1

Advanced plane geometry research III: Alternative proofs for the standard theorems in plane geometry

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The proofs of theorems which have been reputed in the history of plane geometry since the time of ancient mathematicians, particularly geometers, have not been further analysed and as a result it has not been possible to discover some alternative proofs for those theorems due to lack of research interest and the complexity of plane geometry particularly in Sri Lanka and possibly in other countries. Composing an alternative proof for a theorem in plane geometry is conducive of being discovered another relationships which could be mostly significant towards the development of geometry as plane geometry relationships or theorems have been lacking in recent years and even at present. Therefore, this paper attempts to provide alternative proofs for the standard theorems in plane geometry.

- (1) ABCD is a cyclic quadrilateral as $AB=a, BC=d, CD=c, AD=b$. The theorem is that $AC \cdot BD = AB \cdot DC + AD \cdot BC$. This is called as the Ptolomy theorem regarding cyclic quadrilaterals. The alternative proof of this theorem is based on 3 perpendiculars being drawn to DC, AC and DA which are composed in order to obtain the multiplication of AC and BD through similar triangles. An extra considerable result is achieved in the research from the similarity of natural triangles bounded in ABCD cyclic quadrangle as $AC/BD = (ab+cd)/(ad+bc)$ which is the division of the diagonals of a cyclic quadrangle. Then by using those results it can be obtained the reputed lengths of two diagonals from a, b, c and d.
- (2) ABC is any triangle. AD is the median of the triangle which meets BC at D. The theorem is that $AB^2 + AC^2 = 2BD^2 + 2AD^2$ which is called as the Apollonius theorem. The alternative proof of this theorem is based on the similar triangles which are emerged by the circum circle of ABC triangle and the use of the Ptolomy theorem mentioned earlier. The most significant fact in this research is that proving Apollonius theorem without any use of the Pythagoras theorem which exposes that Apollonius theorem is not a derivative of Pythagoras theorem.
- (3) ABC is any triangle. AD is the internal bisector of the angle A which meets BC at D. The theorem or the relationship is that $AD^2 = AB \cdot AC - BD \cdot DC$ which denotes the length of the internal bisector of ABC triangle. The alternative proof of this theorem is based on a similar triangle which is constructed between AC and BC lines.