3.11 Periodical outbreak of Tuberculosis Epidemic: An epidemic model with dynamic host population

K.H.P. Madhuwanthi[1] , N.G.A. Karunathilake[2]
Department of Mathematics, University of Kelaniya, Kelaniya,

ABSTRACT

1 Introduction

In this work we develop a mathematical model to study the mechanism by which the Tuberculosis spread, to predict the future course of outbreak of the disease and evaluate the strategies to control the epidemic. The model consider the major tuberculosis prevention programme in Sri Lanka, that is vaccine BCG (Bacillus Chalmette Guerin) all newborn immediately after the birth. The model takes into account the fact that the BCG is not a lifelong immunity and it supplies only five year immunity. In many epidemic models it is assumed that the birth rate and the death rate of the host population are the same and thereby the host population remains unchanged [1], [2]. However, in our consideration the dynamic behavior of the host population is taken into account. A detailed analysis is carried out to investigate the existence and the uniqueness and the stability of the solution of the model. The parameters of the model are estimated using the recent fast data issued by the Department of Census and Statistics, Sri Lanka [4].

2 SIRVN for the description of the tuberculosis

A deterministic compartment model SIRVN is based on the epidemiological behavior of the tuberculosis in the human body. The host population (N) is subdivided into a set of distinct compartments, namely, Susceptible (S), tuberculosis infected (I), recovered (R) and vaccinated (V). Considering the rate of flux of individuals between compartments and the dynamic of the population, the following system of ordinary differential equations can be derived for the description of the dynamic of S, I and V, respectively.

$$\frac{dS}{dt} = b_2 V - dS - \beta\frac{SI}{N}, \qquad \frac{dI}{dt} = \beta\frac{SI}{N} - dI - \gamma I,$$

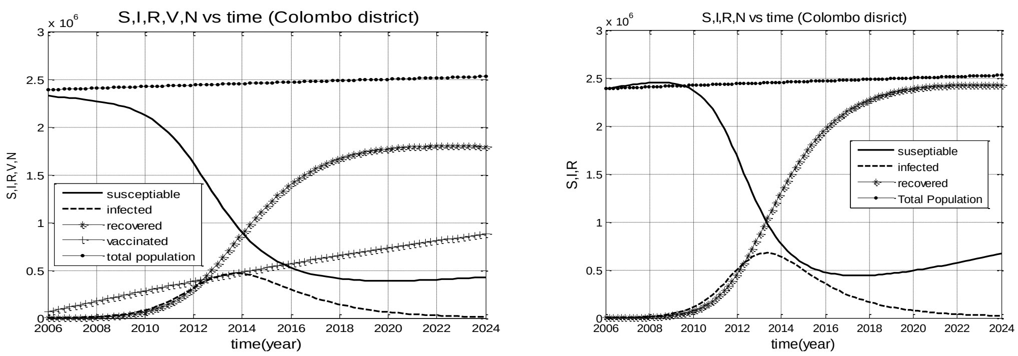
$$\frac{dR}{dt} = \gamma I - dR, \qquad \frac{dV}{dt} = b_1 N - b_2 V - dV,$$

$$\frac{dN}{dt} = (b_1 - d)N\left(1 - \frac{K}{N}\right).$$

Here, b_1, b_2, d, K , β and γ are the birth rate, vaccine rate, death rate, carrying capacity, contact rate with susceptible and recovery rate, respectively. The existence and uniqueness of the solution of the system is established as an immediate consequence of the conditions of the Picard-Lindelof theorem. The stability of two equilibrium points of the system, namely, the disease free state (DFE) and the endemic state (EE) of the epidemic, is also investigated through the sign of the eigenvalues of the Jacobian Matrix at the equilibrium points. Birth rates and death rates are obtained from the annual reports and the parameters β and γ are estimated by using the shooting algorithm for the data of all individuals in each compartment [1].

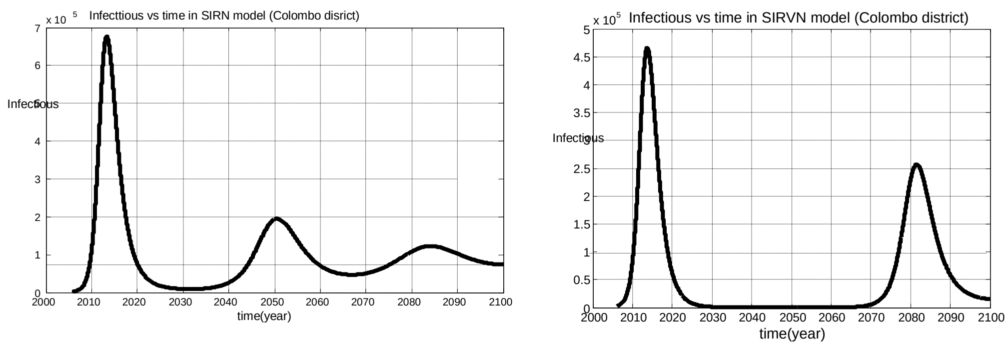
5 Simulation Results and Concluding Remarks.

Analysis of the model reveals that the dynamic of the tuberculosis can be successfully describe using the proposed compartment epidemic model. In order to investigate the ability of the vaccine programme in controlling the tuberculosis epidemic, we compare the simulation results of SIRN and SIRVN models.



5.1 Figure

The simulation result shows that in the absence of vaccination programme, most of individuals in susceptible compartment are moved in to the infected compartment at a higher rate (see Figure 5.1). However, the vaccination programme decreases the rate of movement of susceptible individual into the infectious compartment. In both situations, the absence of vaccine programme and the presence of vaccine programme there will be periodical epidemic states of tuberculosis (see Figure 5.2). In the absence of vaccination programme, the periodic time is about 30 years. The vaccination programme delays this periodic time by another 30 years as in SIRVN model the periodic time is about 60 years.



5.2 Figure

The carrying capacity of the logistic model for the description of the population depends on the resource in the region. The qualitative behavior does not change for different value of carrying capacity.

References

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