3.1 Path of a light ray near a body with cosmological constant

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ABSTRACT

Emitted light rays from a very distant and bright source are deflected between the source and the observer when they pass near a massive body with an enormous gravity. As a result the massive body such as a cluster of galaxies have an ability to perform as a gravitational lens. In recent times, some authors [1] have found that the cosmological constant Λ, affects the phenomenon of gravitational lensing.

In this paper, we have corrected an expression for the total deflection angle which was published in 2008 of our first paper regarding this subject [2] . Considering the effect of the cosmological constant, we have also found two equations for the path of a light ray when it passes near a massive object with a very high gravitational influence.

We consider the Schwarzschild-de Sitter metric,

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

that represents the space time outside a body of mass M, when the cosmological constant is taken into consideration,

where $$f(r) \equiv 1 - \frac{2m}{r} - \frac{\Lambda r^2}{3} \text{ [3]},$$

and we use relativistic units $(c = G = 1)$.

The null-geodesic equation in Schwarzschild-de Sitter metric can be written as

$$E^2 - h^2u'^2 - h^2u^2 + 2mh^2u^3 + \frac{\Lambda h^2}{3} = 0, \tag{1}$$

where E is the energy, h is the orbital angular momentum and $u = \frac{1}{r}$.

When Λ is small, the total deflection angle, 2δ is obtained as,

$$2\delta = 4\frac{m}{r_0} + \frac{15\pi}{4}\frac{m^2}{r_0^{\,2}} - \frac{45\pi}{6}\frac{m^4}{r_0^{\,4}} + \left(-\frac{2r_0}{3}m - \frac{5\pi}{8}m^2 + \frac{4}{3r_0}m^3 + \frac{5\pi}{r_0^{\,2}}m^4 \right)\Lambda \qquad (2)$$

by neglecting fifth and higher order terms of ε, where $\varepsilon = 3m$ and r_0 is the closest approach of the light ray from the centre of the massive body.

Recently published papers [1], [3] have expressions for the total deflection angle involving $\frac{1}{m}$ term. However, we do not expect the deflection to tend to infinity as m tends to zero. In contrast our expression gives a different form as from equation (2) 2δ approaches zero when $m \to 0$,. Equation (2) also agrees with the expression for the deflection in the Schwarzschild metric when Λ is neglected.

Taking the limiting case of $m = 0$ in equation (1),

$$\left(\frac{du}{d\phi}\right)^2 = \frac{E^2}{h^2} + \frac{\Lambda}{3} - u^2 .$$

By integrating, we have also obtained two solutions of the above equation that can be expressed as follows.

$$\text{When } r > \frac{1}{\left(\frac{E^2}{h^2} + \frac{\Lambda}{3}\right)^{\frac{1}{2}}} ;\ u = \left(\frac{E^2}{h^2} + \frac{\Lambda}{3}\right)^{\frac{1}{2}} \sin(\phi + A)$$

$$\text{When } r < \frac{1}{\left(\frac{E^2}{h^2} + \frac{\Lambda}{3}\right)^{\frac{1}{2}}} ;\ u = \left(\frac{E^2}{h^2} + \frac{\Lambda}{3}\right)^{\frac{1}{2}} \cosh(\phi + B)$$

where *A* and *B* are constants.

References

[1] Rindler, W., Ishak, M. '*The Contribution of the Cosmological Constant to the Relativistic Bending of Light Revisited*', [astro-ph]: arXiv: 0709.2948v1 (2007).

[2] Jayakody, J.A.N.K., de Silva, L.N.K. '*Deflection of light with cosmological constant*', 9th Annual Research Symposium 2008, University of Kelaniya (2008).

[3] Ishak, M., Rindler, W., Dossett, J., Moldenhauer, J., Allison, C. '*A New Independent Limit on the Cosmological Constant/Dark Energy from the Relativistic Bending of Light by Galaxies and Clusters of Galaxies'*, [astro-ph]: arXiv: 0710.4726v1 (2007).