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Numerical solutions of fuzzy differential equations using higher order methods

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Fuzzy differential equations (FDEs) are useful for modelling real-world problems in science and engineering and have been studied by many researchers. Fuzzy solutions are required for certain problems due to the inherent uncertainty of the initial conditions. For instance, Fuzzy Predator Prey (FPP) models and fuzzy RLC-circuit models. The predator-prey model and RLC-Circuit model consist of a system of first-order differential equations. When the initial conditions are imprecise, they can be modelled as triangular Fuzzy numbers. In this study, four fuzzy numerical methods: namely, fuzzy Euler method, 2nd and 3rd-order fuzzy Taylor methods, and 4th-order fuzzy Runge-Kutta method were used. Those methods were implemented and validated for the solutions of certain Fuzzy Predator Prey (FPP) models and Fuzzy RLC-circuit models using MATLAB. The absolute and relative errors were compared with the analytical solution when available and among the methods. It was observed that the 3rd-order fuzzy Taylor method gives a better approximation (and minimum error) for the models. In addition, stability analysis, qualitative error analysis, sensitivity analysis of the crisp parameters, and fuzzy initial populations of the above FPP model were studied. From the stability analysis, it was observed that there is a fuzzy stable equilibrium point for the FPP model. According to the sensitivity analysis of the crisp parameter in the FPP example, it can be concluded that when the natural growth rate increases, the prey population increases, and the maximum number of predators also increases in the shorter period, as the death rate per encounter of preys due to predation increases the maximum number of prey and predators decrease. The first peak in predators occurs sooner, and as the natural death rate of predators in the absence of the prey increases, the maximum number of prey increases, and as the death rate goes to zero, the predator population remains constant. When the reproduction rate of predators for each prey captured increases, the prey population decreases, and it does not show a significant variation in the peak of the predator populations. By the sensitivity analysis of the initial population in the FPP model, it was observed that the maximum number of preys is larger than the maximum number of predators and the fuzzy phase plane is closed or open. Also, it can be concluded that as in the unfuzzy version of the algorithms, there's a trade-off between step size, accuracy, and computational cost. In particular, if the step size is too large, the solution fails to converge.

Keywords: Fuzzy Differential Equations, Fuzzy Predator-Prey model, Fuzzy RLC-Circuit model, Fuzzy Taylor Methods, Fuzzy Runge-Kutta method.