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Paper: Diversity

Visualization of dihedral groups and their subgroups

A picture may be worth a thousand words; therefore most of the mathematicians use images to demonstrate mathematical concepts like Riemann sums, Topological surfaces, graphs, etc. Then why do they not use images to demonstrate groups as well? This is where the area of visualization of group theory comes into play. For this purpose, the most significant method was invented by the British mathematician Arthur Cayley and called it Cayley diagrams. There are other visualization techniques such as multiplication tables, object with symmetries, etc. However, a Cayley diagram has two major advantages compared to other methods. First, it clearly represents a group as a set of elements and the second is by taking the group's generators into account. Cayley diagrams fully illustrate the interaction among the elements of a group.

In our work, we mainly focused on drawing diagrams of Dihedral groups (D_n) to illustrate order of each element, inverse of every element and their positions in the diagrams. Furthermore, we determined all subgroups without any calculations. In this study, we divided *n* in to two cases as odd and even. We have used

$$D_n = \{a, b \mid a^n = e, \ b^2 = e, \ bab = a^{-1}\}$$

the Dihedral group of order *2n*.

Using the idea in Cayley diagrams, we have drawn a diagram for D_n., for which we use two *n*-gons one within the other. Starting from the identity element, we give the correct positions for all the elements of D_n. Inner *n*-gon contains order *2* elements only and the outer *n*-gon contains the other elements with different orders with an interesting relationship between the divisors of *n*. After drawing the required diagram, we have discussed the orders of all the elements of D_n just by looking at it. Also, we can find the positions and inverse of each element by folding the diagram through its symmetric axis. The number of subgroups of a Dihedral group D_n is given by the formula, $d(n) + \sigma(n)$, where $d(n)$ is the number of divisors of *n* and $\sigma(n)$ is the sum of the divisors of *n*.

Using the relation in between the divisors of *n* and the order of each element of D_n , we found all subgroups of D_n just by looking at the diagram.

Our main aim was to find a graphical method which is easier than using formulas to explain Dihedral group of any order. We concluded that any given Dihedral group D_n, the order of each element, inverses of each element and their positions and all its subgroups can be derived easily by visualization through an appropriate Cayley diagram.