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Modelling and Forecasting the Volatility of Daily Exchange Rate Using GARCH Model

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***Abstract*— The volatility of daily exchange rate is a significant economic indicator for open economic countries like Sri Lanka, when considering the international level trade. Therefore, it is very important to be aware of the behavior of future fluctuations of the exchange rate volatility, even though accurate volatility forecasting is challenging. The purpose of this study is to model and forecast the volatility of the US Dollar against the Sri Lankan Rupee (USD/LKR) daily exchange rate. The daily USD/LKR exchange rate data from 1st January 2015 to 30th April 2021 were used in this study and it was found that the exchange rate was continuously increasing throughout the period and stationarity of the daily exchange rate return series was confirmed by the Augmented Dickey-Fuller (ADF) test. Volatility of the daily exchange rate returns were modeled using Generalized Auto-Regressive Conditional Heteroscedastic (GARCH) models. ARMA(2,1) was found to be the most preferable model for the mean equation, and GARCH(1,1) was identified to be best to capture the conditional volatility of the residuals of the ARMA(2,1) model. In addition, Lagrange Multiplier (LM) test clearly showed that ARCH effect no longer exists in the residuals of ARMA(2,1) - GARCH(1,1) model and the Sign Bias test indicated that there was no asymmetry effect in residuals. Therefore ARMA(2,1) - GARCH(1,1) was identified as the best model to forecast the USD/LKR daily exchange rate with Mean Squared Error (MSE), Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) equal to 0.047695, 0.218391, and 0.047695 respectively. The findings of this study can be used in decision and policy making stages to minimize the risk associated with exchange rate volatility.**

Keywords — Exchange rate, GARCH, Volatility

I. INTRODUCTION

From the earliest stages of globalization, many countries tended to international trade where the transactions mostly depend on the USD and the foreign exchange market has been expanded with time. The foreign exchange rate is the relation of the value of two currencies which is regarded as the relative value. Volatility of the exchange rate is indicated by the relation of the demand and supply of the currency in the foreign exchange market. Therefore exchange rate is a crucial economic factor to continue the economic stability of all countries and its fluctuation significantly impacts the macroeconomic variables like gap between net exports and imports, interest rate, debt payments installment of the countries, inflation, unemployment, etc. However the global economic crisis leads to the volatility of the exchange rate fluctuations which mainly affect the economic activities of the developing open economy countries like Sri Lanka than other developed countries.

Modeling and forecasting of exchange rate volatility are great interest to decision and policy making at different levels to minimize the risk associated with exchange rate volatility. Most researchers have studied the non-constant variance in the financial time series like exchange rate, and uncertainties in prices and returns. Therefore, the behavior of exchange rate volatility is explained using time series econometric models. A study on analyzing daily closing prices for four currency pairs viz. Euro, Pound, Swiss Franc and yen against the US Dollar [1], using data from January 2002 to December 2011 forecasted the volatility of each foreign exchange rate by using univariate GARCH models. It was compared the in-sample forecasts from symmetric and asymmetric GARCH models with the implied volatility derived from currency options for each dollar parity. A low volatility was noted during the period of 2002 to 2007, and there was high volatility during the rest of the period. Finally, it was concluded that the volatility forecasts significantly outperform the GARCH models in both low and high volatility periods.

Univariate nonlinear time series models such as Auto-Regressive Conditional Heteroscedastic (ARCH) and GARCH models have been used to examine the behavior of the daily (TZS/USD) exchange rate [2] by using data from January 2009 to July 2015. In the same study, Exponential GARCH (EGARCH) model also has been used to capture the asymmetry in volatility clustering and the leverage effect in exchange rate. In a study of identifying the volatility of the RM/Sterling exchange rate by using GARCH models [3], the maximum likelihood method and several goodness-of-fit statistics were used to estimate the parameters of these models and diagnosed the performance of the within-sample estimation. Also the accuracy of the out-of-sample and one-step-ahead forecasts were evaluated using mean squared error. In this study the stationary GARCH-M outperforms other GARCH models in out-of-sample and one-step-ahead forecasting. When using random walk model as the naive benchmark, all GARCH models outperform this model in forecasting the volatility of the RM/Sterling exchange rate.

To understand the theoretical and empirical performance of the GARCH class of models and to exploit the potential gains in modeling conditional variance, a study was carried out to forecast monthly exchange rates of Pakistan for the period ranging from July 1981 to May 2010[4]. ARMA, ARCH, GARCH, and EGARCH models were used in this study and found that GARCH (1, 2) was best to remove the persistence in volatility while EGARCH (1, 2) successfully overcome the leverage effect in the exchange rate returns. When it comes to the context of Sri Lanka, the behavior of the daily USD/LKR exchange rate was examined by using several ARCH and GARCH models[5], and found that ARMA(1,1) – ARCH(6) is the best fitted model to forecast the daily USD/LKR Exchange rate volatility.

Even though above researchers have studied about forecasting exchange rates in various contexts, with the dynamic nature of the volatility of exchange rate, it is vital to study further and identify better forecasting models to forecast exchange rate and it will assist to reduce the risk associated with investment and policy making decisions. Therefore, this research is focused on identifying a forecasting model to forecast USD/LKR daily exchange rate volatility using GARCH model.

II. OBJECTIVES

The main objective of this research is to analyze the behavior of the daily USD/LKR exchange rate over the study period and to construct forecasting (ARCH family) model to forecast the exchange rate while evaluating the forecasting performance of the best fitted model.

III. METHODOLOGY

The data used in this study consists of daily exchange rates of USD/LKR from 1st January 2015 to 30th April 2021 obtained from the Central Bank of Sri Lanka which comprises 1520 observations. Exchange rate return was calculated from the following Equation (1):

$$R_t = ln\left(\frac{X_t}{X_{t-1}}\right) = ln\, ln\,(X_t) - ln(X_{t-1}) \quad (1)$$

, where R_t is the daily return of USD/LKR exchange rate at time t and X_t is the daily USD/LKR exchange rate at time t.

The data set was divided into two sets namely training and testing where the training set ranging from 1st January 2015 to 31th May 2019 (1062 observations) and the testing set ranging from 3th June 2019 to 30th April 2021 (458 observations).

First, the stationarity of the return series of daily exchange rate was checked by using Augmented Dickey-Fuller (ADF) test [6]. Box Jenkin's methodology was applied next to model the conditional mean equation of the exchange rate return series [7]. The combination of AR(p), MA(q) and ARMA (p, q) models were used and latter is expressed in Equation (2):

$$R_t = c + \sum_{i=1}^{p} \varphi_i R_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \quad (2)$$

, where φ_i and θ_i are parameters, c is a constant, and ε_t is the residual at time t. The best model for the mean equation was selected by comparing the accuracy measures such as Akaike Information Criterion (AIC), Bayesian Information Criteria (BIC), Durbin-Watson (DW) statistic and R-square value of identified adequate models. The residual diagnostics of the mean equation were tested by correlogram Q-statistic and Jarque-Bera test. The presence of ARCH effect of the residuals of the conditional mean model was tested by using the Lagrange Multiplier (ARCH-LM) test [8]. The conditional variance of the residuals of the conditional mean model was modeled by using two time-varying volatility models namely ARCH and GARCH. Engle (1982) [9] developed ARCH model to capture the volatility of the financial time series. The general ARCH(q) model assumes normally distributed residuals where the current conditional variance depends on the first q previous squared innovations as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \quad (3)$$

, where $\omega > 0, \alpha_i \geq 0$ *and* $i > 0$.

Bollerslev (1986) [10] developed the GARCH model and the general GARCH (p, q) model assumes normally distributed residuals where the current conditional variance depends on the first p previous conditional variances and the first q previous squared innovations as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (4)$$

, where $\omega > 0, \alpha_i \geq 0, \beta_j > 0$ *and* $i, j > 0$.

The best model for the variance equation was selected by comparing the accuracy measures such as AIC, BIC, DW statistic and R-square value of identified adequate models. Then, the ARCH-LM test was conducted to test the ARCH effect of the residuals of selected conditional variance model and also the serial correlations, normality and asymmetric effect of residuals were tested. Finally, the forecasting performance of the best fitted model was evaluated by using MSE, RMSE and MAE.

IV. RESULTS AND DISCUSSION

Fig. 1. The time series plot of the daily USD/ LKR exchange rate

Figure 1 shows the time series plot of the daily USD/LKR exchange rate which gives a comprehensive knowledge about the daily USD/LKR exchange rate variation during the past six years. From 2015 to 2021, the exchange rate has been increased dramatically from Rs. 130 to Rs.200. The global economic crisis due to Covid-19 pandemic is the main reason for the unexpected increment of the exchange rate during last year.

A forecasting model can be fitted only for a stationary time series. The stationarity of the exchange rate return series was tested by using ADF test based on the following hypothesis and the results are shown in Table 1.

H_0: return series is not stationary
H_1: return series is stationary

Table 1. ADF test

		T - Statistic	**Prob.***
Augmented Dickey – Fuller test statistic		-40.93181	0.0000
Test critical values:	1% level	-3.434454	
	5% level	-2.863240	
	10% level	-2.567723	

According to Table 1, the null hypothesis is rejected as the p-value of the ADF test statistic (0.000 < 0.05) is significant at 5% significance level and it can be concluded that the daily exchange rate return series is stationary. Then, several tentative conditional mean models were identified by using Box Jenkin's methodology and the best mean equation was identified by comparing AIC, BIC, DW and R-square values. The accuracy measures of the adequate mean models are shown in Table 2.

Table 2. Accuracy measures of adequate mean models

Model	**AIC**	**BIC**	**R^2**	**DW**
AR(1)	-9.026841	-9.017471	0.015821	1.984824
MA(1)	-9.023793	-9.014423	0.012811	2.023697
AR(2)	-9.027237	-9.013182	0.01807	2.003955
MA(2)	-9.028238	-9.014183	0.019054	1.995333
ARMA(1,2)	-9.037846	-9.019106	0.030457	2.017056
ARMA(2,1)	-9.039877	-9.021138	0.032424	1.996509

Table 2 indicates the results of the accuracy measures of the identified tentative conditional mean models. By comparing the AIC, BIC, R-square and DW statistics of each model, ARMA (2,1) model was selected as the best conditional mean model and the corresponding equation is as follows.

$$R_t = 0.791211\, R_{t-1} + 0.190915\, R_{t-1} - 0.962305\, \varepsilon_{t-1} \quad (5)$$

The serial correlation of the residuals of mean equation was tested using the correlogram Q-statistic and the results are shown in Figure 2.

H_0: There is no autocorrelation in the residuals
H_1: There is autocorrelation in the residuals

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.001	-0.001	0.0004	
		2	0.008	0.008	0.0634	
		3	0.003	0.003	0.0705	
		4	0.014	0.014	0.2805	0.596
		5	-0.007	-0.007	0.3380	0.845
		6	-0.028	-0.029	1.1969	0.754
		7	-0.026	-0.026	1.9354	0.748
		8	0.003	0.004	1.9481	0.856
		9	-0.001	-0.000	1.9488	0.924
		10	-0.050	-0.050	4.6733	0.700
		11	-0.057	-0.057	8.1116	0.423
		12	-0.032	-0.033	9.1786	0.421
		13	0.026	0.026	9.9091	0.449
		14	0.019	0.021	10.312	0.503
		15	0.024	0.025	10.938	0.534
		16	0.031	0.028	11.944	0.532
		17	0.027	0.020	12.706	0.550
		18	-0.005	-0.009	12.729	0.623
		19	-0.001	-0.002	12.731	0.692
		20	-0.004	-0.005	12.753	0.753
		21	-0.005	-0.009	12.783	0.804
		22	0.021	0.018	13.267	0.825
		23	0.011	0.013	13.407	0.859
		24	-0.003	0.002	13.415	0.893
		25	-0.010	-0.004	13.521	0.918

Fig. 2. The Correlagram Q-Statistic of ARMA (2, 1) model

All p values are greater than 0.05 according to Figure 2 and null hypothesis is not rejected at 5% level of significance confirming the non-existence of serial correlation in residuals of the best fitted conditional mean model. The normality of the residuals was tested based on the following hypothesis [11].

H_0: Residuals are normally distributed

H_1: Residuals are not normally distributed

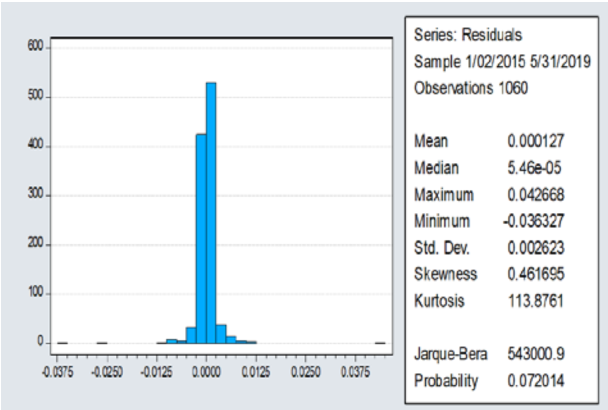


Fig. 3. The histogram of the residuals of ARMA (2, 1) model

According to Figure 3, it is clear that the Jarque-Bera test statistic is not significant (0.07 > 0.05) at 5% significance level and null hypothesis is not rejected, confirming the normality of residuals. ARCH-LM test was used to check the homoscedasticity of residuals using the following hypothesis and based on the results of Table 3, it can be concluded that there is an ARCH effect in the residuals as the p-value of the ARCH - LM test (0.0000 < 0.05) is significant at 5% significance level and the null hypothesis is rejected.

H_0: There is no ARCH effect in the residuals
H_1: There is an ARCH effect in the residuals

Table 3. LM test on ARMA (2, 1) residuals

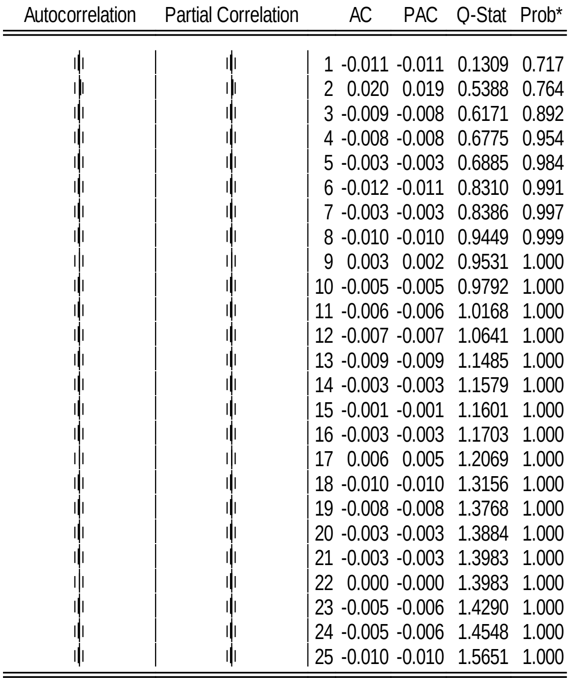
F - statistic	268.7309	Prob.F(1,1057)	0.0000
Obs* R-squared	214.6635	Prob. Chi-Square(1)	0.0000

As the residuals of the mean equation are heteroscedastic, ARCH and GARCH models were used to estimate the variance equation. The best model for the variance equation was selected by comparing the accuracy measures such as AIC, BIC, DW statistic and R-square value of adequate models and the results are summarized in Table 4.

Table 4. Accuracy measures of adequate models

Model	**AIC**	**BIC**	**R^2**	**DW**
ARMA(2,1) - ARCH(1)	-9.750007	-9.726613	-1.652382	3.36569
ARMA(2,1) - GARCH(1,1)	-10.05404	-10.02589	0.012868	1.98407

Based on the results of Table 4, the accuracy measures of tentative models were compared and ARMA(2,1) – GARCH (1,1) model was selected as the most preferable model for the variance equation.



	AC	PAC	Q-Stat	Prob*
1	-0.011	-0.011	0.1309	0.717
2	0.020	0.019	0.5388	0.764
3	-0.009	-0.008	0.6171	0.892
4	-0.008	-0.008	0.6775	0.954
5	-0.003	-0.003	0.6885	0.984
6	-0.012	-0.011	0.8310	0.991
7	-0.003	-0.003	0.8386	0.997
8	-0.010	-0.010	0.9449	0.999
9	0.003	0.002	0.9531	1.000
10	-0.005	-0.005	0.9792	1.000
11	-0.006	-0.006	1.0168	1.000
12	-0.007	-0.007	1.0641	1.000
13	-0.009	-0.009	1.1485	1.000
14	-0.003	-0.003	1.1579	1.000
15	-0.001	-0.001	1.1601	1.000
16	-0.003	-0.003	1.1703	1.000
17	0.006	0.005	1.2069	1.000
18	-0.010	-0.010	1.3156	1.000
19	-0.008	-0.008	1.3768	1.000
20	-0.003	-0.003	1.3884	1.000
21	-0.003	-0.003	1.3983	1.000
22	0.000	-0.000	1.3983	1.000
23	-0.005	-0.006	1.4290	1.000
24	-0.005	-0.006	1.4548	1.000
25	-0.010	-0.010	1.5651	1.000

Fig. 4. The Correlogram of square residuals of ARMA (2, 1) – GARCH (1,1) model

H_0: There is no autocorrelation in the residuals
H_1: There is autocorrelation in the residuals

The serial correlation of the residuals of variance equation was tested using the correlogram Q-statistic of the squared residuals and the results are shown in Figure 4.

All p values are greater than 0.05 according to Figure 4 and null hypothesis is not rejected at 5% level of significance confirming the non-existence of serial correlation in residuals of the variance equation. ARCH-LM test was used to check the homoscedasticity of residuals and based on the results of Table 5, it can be concluded that there is no ARCH effect in the residuals as ARCH - LM test statistic (0.7183 > 0.05) is not significant at 5% significance level and the null hypothesis is not rejected.

H_0: There is no ARCH effect in the residuals
H_1: There is an ARCH effect in the residuals

Table 5. LM test on ARMA (2, 1) - GARCH(1,1) residuals

F - statistic	0.130202	Prob.F(1,1057)	0.7183
Obs* R-squared	0.130433	Prob. Chi-Square(1)	0.7180

Asymmetric effect of the conditional variance model was tested using following hypothesis [12] and the results are shown in Table 6.

H_0: There is no asymmetric effect
H_1: There is an asymmetric effect

Table 6. Sign Bias test results

Sign Bias Test	t - value	prob. sig.
Sign Bias	1.147660	0.2512
Negative Sign Bias	0.002899	0.9977
Positive Sign Bias	0.020700	0.9835
Joint Effect	1.426738	0.6993

According to Table 6, it is clear that the null hypothesis is not rejected at 5% significance level as the p value of Sign Bias test is greater than 0.05. Therefore, it can be concluded that there is no asymmetric effect in residuals of the ARMA(2, 1) - GARCH (1, 1) model.

Table 7. Estimated coefficients of the ARMA(2, 1) - GARCH (1, 1) model

Variable	Coefficient	Prob.
AR(1)	0.719951	0.0000
AR(2)	0.286423	0.0000
MA(1)	-0.969023	0.0000
ω	1.81E-07	0.0000
RESID(-1)^2-(α)	2.153248	0.0000
GARCH(-1)-(β)	0.272999	0.0000

The Estimated coefficients of the ARMA(2, 1) - GARCH (1, 1) model is given in Table 7 and it indicates that the coefficients of both mean equation and variance equation are significant at 5% level of significance. The positive coefficients confirm the non-negative constraints of the model. The significance of α and β indicated that the conditional variance depends on the lagged squared residuals and the lagged conditional variance. That is the news about volatility from previous periods have a significant impact on the current volatility. Finally, forecast performance of the best fitted ARMA(2, 1) - GARCH (1, 1) model was evaluated using MAE, RMSE and MAE and the results are shown in Table 8.

Table 8. Forecast performance of ARMA (2, 1) - GARCH(1,1) model

Accuracy matrix	Value
mean square error (MSE)	2.0457529e-05
root mean square error (RMSE)	0.004525
mean absolute error (MAE)	0.002192

According to the results of Table 8, it is clear that the forecasting accuracy of the ARMA(2,1) - GARCH(1, 1) model is excellent as MAE, RMSE and MAE are very close to 0.

V. CONCLUSION

This study was focused on building a forecasting model to forecast the daily USD/LKR exchange rate returns using GARCH model. The stationarity of the daily exchange rate return series was examined using ADF test. ARMA(2,1) model was identified as the best mean equation of the exchange rate returns based on the results of AIC, BIC, DW statistic and R-square value. The ARCH-LM test confirmed the presence of ARCH effect in the residuals of the conditional mean equation. After comparing accuracy measures of adequate ARCH and GARCH models, GARCH(1, 1) model was identified as the adequate model to capture the remaining conditional heteroscedastic effect of the mean model. Moreover, the ARCH-LM test was performed to test the additional ARCH effect in the residuals of ARMA(2,1) - GARCH(1, 1) model and resulted with no ARCH effect. Further, Sign-Bias test confirmed that there is no asymmetric effect in the residuals of ARMA(2,1) - GARCH(1, 1) model. Therefore ARMA(2,1) - GARCH(1, 1) model was identified as the best model to forecast the volatility of USD/LKR exchange rate return series. Finally, the forecast performance of the identified model was evaluated and MSE, RMSE and MAE were found to be to 0.047695, 0.218391, and 0.047695 respectively. Therefore it can be concluded that the prediction accuracy of the ARMA(2,1) - GARCH(1, 1) model is ample to make better decisions to minimize risk associated with USD/LKR exchange rate volatility.

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