Oral presentation: 142

Explicit and exact traveling wave solutions for generalized Benjamin-Bona-Mahoney-Burgers equation with dual high-order nonlinear terms

M. T. F. Hasna* and T. Mathanaranjan

Department of Mathematics and Statistics, Faculty of Science, University of Jaffna, Sri Lanka
*hasnathahir8605@gmail.com

In recent years, the investigation of exact solutions to nonlinear partial differential equations has played an important role in nonlinear phenomena. Several powerful methods have been proposed to obtain exact solutions of nonlinear partial differential equations, such as the first integral method, tanh-sech method, sine-cosine method, Jacobi elliptic function method, F-expansion method, exp-function method, (G'/G)-expansion method and so on. The first integral method was first proposed by Feng in solving Burgers-KdV equation. It is a direct algebraic method based on the commutative algebra. Recently, it was successfully used for constructing exact solutions to a variety of nonlinear problems. Our interest in the present work is in implementing the first integral method to find the exact solution for generalized Benjamin-Bona-Mahoney-Burgers (BBMB) equation with dual high-order nonlinear terms. Considering the generalized BBMB equation with dual arbitrary power-law nonlinearity

$$\frac{\partial u(x,t)}{\partial t} + \alpha \frac{\partial u}{\partial x} + \left(\beta u^n + \gamma u^{2n}\right)\frac{\partial u}{\partial x} + \tau \frac{\partial^2 u}{\partial x^2} + \delta \frac{\partial^3 u}{\partial x^2 \partial t} = 0, \quad (1)$$

where α represents the strength of a defection or drifting, β and γ are the coefficients of the dual power-law nonlinearity with the exponent n, τ and δ are the dispersion coefficients. Thus, Eq. (1) models the shallow water wave flow on lakes and beaches with a dissipative factor that is given by the coefficient τ. For $n = 1, \beta = 0$ and $\delta = -1$, Eq.(1) reduces to the classical BBMB equation and for $\tau = 0$, Eq.(1) reduces to the generalized Benjamin-Bona-Mahoney (BBM) equation which have been discussed using several techniques. In this study, the first integral method is applied to solve the BBMB equation with dual high-order nonlinear terms. As a result, we obtain explicit and exact traveling wave solutions $u(x,t) = \phi(\xi), \xi = k(x - ct)$, where k and c are arbitrary constants of the form

$$\phi(\zeta) = \pm\left\{\frac{1}{4}\sqrt{\frac{(1+n)(1+2n)}{c\gamma\delta}}\left(-\tau + \sqrt{4c(-c+\alpha)\delta + \tau^2}\right)\left(-1 + \tanh\left[\frac{n(-\tau + \sqrt{4c(-c+\alpha)\delta + \tau^2})}{4ck\delta}(\zeta + \zeta_0)\right]\right)\right\}^{\frac{1}{n}}.$$

As mentioned above, for some specific choices of parameters, our exact solutions reduce to the solutions of classical BBMB equation and BBM equation which are exactly the same result derived by the tanh method and (G'/G)-expansion method. Moreover, we plot three dimensional graphics of our exact solutions which denote the vitality of the solutions. These new explicit and exact solutions may be important for the explanation of some practical physical problems.

Keywords: BBMB equation, BBM equation, first integral method, traveling wave solutions