

A mathematical model for a lubricant approximation of the wet thin tear film

G. P. Ranathunga* and N. G. A. Karunathilake

*Department of Mathematics, Faculty of Science,
University of Kelaniya, Sri Lanka
Email: gpranathunga@gmail.com*

The classical description of the tear film resides on the anterior surface of the eye between the upper and lower lids is a wet thin film. Various fluid dynamic model have been developed for the evolution of the surfactant concentration and the thickness of precorneal thin tear film on the eye surface after each blink. In this work we model tear film as an incompressible Newtonian fluid together with the surfactant equations with appropriate boundary conditions. On a lubricant framework we formulate the motion of the tear film mathematically using mass, momentum and transport equations with free surface boundary conditions. The conjoining pressure in the film is modelled by the standard Van der Waals force with Hamaker constant. The non-dimensional model is discretized using Finite volume method together with nonlinear multigrid approach. This Multigrid approach to the mathematical model with the conjoining pressure improves the results of the model. Study reveals that near the lower lid the thickness comes down from the initial condition but subsequently it advances to reach a maximum at somewhere around the middle and gradually decreases to its equilibrium level to the end. The surfactant concentration in contrast drops steady to zero from lower lid to upper lid. Several dry spots resulting from the evaporation can be observed in the numerical results.

Keywords: Lubricant equation, Newtonian fluid, Surfactant equation, Tear film

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