Analyticity of Elastic S-matrix element for realistic potentials with the Coulomb potential

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ABSTRACT

The analytic properties of the wave function and S-matrix element related to elastic scattering is studied using the Coulomb wave functions and Whittaker functions. It is found that the elastic S-matrix element for local energy independent complex optical potential with the Coulomb potential is analytic in the whole complex $k-$plane except at its poles and two essential singularities, one at the origin and the other at infinity in the upper – half plane. In case of a potential with the Coulomb potential, two S-matrix elements are defined in the scattering theory, the nuclear S-matrix element and the total S-matrix element. It is found that there are infinite number of zeros and poles on the imaginary axis in the nuclear S-matrix element, which is solely due to the Coulomb potential in addition to the essential singularity at the origin. It is found that the both S-matrix elements have an essential singularity at infinity in the upper half – plane as in the case of real potential without the Coulomb potential.

In the study of integral equation method for the potentials without the Coulomb potential, the required upper bounds for the spherical Bessel function and the Neumann function are obtained using elementary mathematics. Using the integral equation, it is shown that the wave function is an analytic function of the wave number k. Elastic S-matrix element is defined by using the asymptotic form of the wave function. It is easily shown that the elastic S-matrix element is unitary for a real potential and also that the elastic S-matrix reaches the value one as k tends to infinity. Other symmetry relations of Elastic S-matrix element are also shown for real and complex potentials without the Coulomb potential.

The upper bounds for the regular Coulomb wave function and the Green function are involved with the integral equation resulting from the Schrödinger equation are derived first. Using the uniform convergence of the integral equation for the wave function, it is found that the wave function is an analytic function of k except at $k = 0$. In this respect it is found that the integral equation for the wave function converges uniformly and hence it is analytic function of k except at $k = 0$. It is also found that S-matrix element for real potential is unitary. However, other symmetry relations are found to be not valid in this case.

Keywords: Analytic, Elastic scattering, S-matrix, Wave function, Poles